

**Evaluation of the**  
**School District of Hillsborough County**  
**Elementary Mathematics Program:**  
*Voyages*

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**August 1, 2007**

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## Curriculum and Factors that Influence its Effective Implementation

The word, curriculum, has different meanings for different individuals. The mathematics community often refers to three types of curriculum: intended, enacted, and achieved (McKnight et al, 1987). We present the meaning of each to clarify the intent of this evaluation report. *Intended curriculum*, as used in this report, refers to the content to be covered as outlined by the 1996 Florida Sunshine State Standards for mathematics and its corresponding Grade Level Expectations (GLEs). The *enacted curriculum* refers to the curricular materials and pedagogical elements that are used by teachers to plan and deliver the intended curriculum. These materials may come from a variety of sources including the textbook, school district resources, and other instructional resources used by the teacher. The *achieved curriculum* refers to that portion of the intended curriculum that is actually mastered by students. The achieved curriculum is dependent on the portion of the intended that is enacted and as a result, learned by students. The achieved curriculum is typically measured by classroom assessments as well as standardized and criterion-referenced assessments such as the Florida Comprehensive Assessment Test (FCAT) that are closely linked to the intended curriculum. These student outcome measures document students' growth of mathematics thinking, knowledge, and abilities over a period of time, typically a year. To fully understand the nature and effectiveness of the curriculum, it is important that each of these types of curriculum is evaluated.

Evaluation of a curriculum is a complex endeavor that requires an understanding of a wide range of human (student and teacher characteristics), curricular (e.g., materials used), and instructional and instructional setting (e.g., school and school district culture and policies) resources. Each aspect represents a complex component to be fully understood. For example, the effectiveness of the enacted curriculum is dependent on several factors that are described below:

- *Professional preparation to deliver the curriculum.* Professional development opportunities that are provided and teacher engagement in such professional development determine the extent to which teachers understand the nature and intent of the intended curriculum and the selected curricula materials to be implemented. In particular, when incorporating a new or innovative curriculum, it is important for teachers to receive professional development prior to implementation that addresses both content and pedagogy, continued support during implementation, and time to reflect on implementation and possible enhancements after implementation. Without such preparation and follow-up support, teachers will not have the understanding needed to deliver the curriculum effectively.
- *Curricula vs. Teacher Philosophy.* Curriculum designers have a particular perspective about the intended use of the curricula materials and the best methods to enhance student knowledge. Teachers also have views related to the best methods for educating students. Teachers who lack understanding about the philosophy that guides the development of curriculum materials or who reject them may ignore the recommended approaches and use other practices and materials that they deem to be more appropriate. Teachers who reject the overriding philosophy of the selected curriculum will not likely implement it with fidelity.

- *Teacher Qualification.* The quality of the teaching force responsible for providing instruction plays a significant role in the implementation of the curriculum. Teacher quality in this case refers to the collective knowledge that teachers possess including knowledge of content, pedagogy, and the learner. The ability of a teacher to use curricula materials effectively depends on his or her ability to facilitate classroom interaction and discourse in ways that enhance students' mathematical development. Teachers with insufficient content knowledge or who lack an understanding of appropriate pedagogy to support student learning will not likely be able to facilitate the development of student knowledge of mathematics or facilitate discussions about specific mathematics topics in ways that enhance conceptual understanding of mathematics.
- *Students' Opportunity to Learn.* Students' opportunity to learn is dependent on the extent to which the intended curriculum is fully addressed. Teachers have great autonomy to make curricula decisions. These decisions influence students' opportunity to learn and the experiences students will have during the learning process. Despite recommended procedures or guides, teachers may skip topics or may spend more time on one topic that results in diminished attention to other topics. Content coverage or the amount of attention provided to particular content topics mediates what content students are exposed to, how they learn that content, and how much they learn.
- *Instructional Quality.* Instructional quality refers to the practices used to deliver instruction. In particular, it focuses on whether or not the pedagogical practices in use support and enhance student learning. Teachers who have a limited pedagogical repertoire or who do not fully understand how to implement the various strategies that are recommended to support learning (e.g., cooperative learning, use of multiple representations, facilitating classroom discourse) may limit the ways in which students learn. That is, they may not be able to use a variety of approaches that allow *all* students to have access and opportunity to learn the subject matter.
- *Students' prior experiences with recommended instructional approaches.* Students may initially be wary of or reject instructional approaches they perceive to be different from the ones they have experienced in the past. Because of this, students may resist new practices causing teachers to experience some difficulty while attempting to implement new approaches. Instruction delivery, during this period, may be difficult because teachers are not only implementing an innovative approach that is unfamiliar, but may also be challenged by student behaviors that on the surface are an attempt to undermine the implementation of a new strategy. Teachers who meet this resistance may not realize that processes need to be incorporated to help students transition to new or different instructional approaches and may simply revert to more familiar practices. However, teachers and students who persevere are able to successfully adapt to new instructional approaches.

- *Assessment Practices.* Assessments are used to measure student learning and guide instructional decisions. Assessments are not limited to those provided as part of the curricular materials, but all practices utilized by the teacher to obtain information about students' learning (e.g., observation, informal interviews, teacher created exams, etc.). The effectiveness of such assessments depends on how they are used, what information they provide, and how teachers interpret and use that information to make instructional decisions. Inappropriate assessments or inappropriate interpretation of assessment results may lead to instructional decisions that may not support student learning.

The factors identified above illustrate that curriculum implementation is actually a complex web of factors that interact concurrently and influence one another. Each factor plays an important role that dictates the extent to which a curriculum can be implemented effectively. These complexities have a greater impact when the selected curriculum represents a departure from practices that are more familiar to school leaders, teachers, students, and parents.

As aforementioned, an examination of the curriculum (intended, enacted, & achieved) is a unique endeavor that necessitates that examination of multiple aspects and influences on the implementation of the curriculum. The National Research Council (2004) provides a framework for the evaluation of curricular effectiveness in its book, *On Evaluating Curricular Effectiveness*. The Council states:

Curricular effectiveness is defined as the extent to which a curricular program and its implementation produce positive and curricularly valid outcomes for students, in relation to multiple measures of students' mathematical proficiency, disaggregated by content strands and disaggregated by effects on subpopulations of students, and the extent to which these effects can be convincingly or causally attributed to the curricular intervention through evaluation studies using well-conceived research designs. Describing curricular effectiveness involves the identification and description of the curriculum according to its programmatic theory and stated objectives; its relation to local, state, and national standards; subsequent scrutiny of its program contents for comprehensiveness, accuracy and depth, balance, engagement, and timeliness and support for diversity; and an examination of quality, fidelity and character of its implementation component. (pp. 36-37)

To engage in such a comprehensive examination of the curriculum would require an analysis of the curricular materials, professional development related directly to the curriculum, teacher quality, teachers' capacity to implement the curriculum, implementation of the curriculum, school factors that may support or hinder implementation, and resultant student outcomes.

We begin by first establishing that the evaluation we were asked to conduct does not represent a comprehensive evaluation of the curricular effectiveness of the elementary mathematics program. Instead, it represents a purposeful and narrow focus on the curriculum as demonstrated by the materials selected to implement the intended curriculum. Specifically, we conducted a *content analysis* of the elementary mathematics program. The conclusions drawn about the curriculum materials are based on our collective experience as mathematics teacher educators at three different institutions of higher education. Collectively, our experiences represent expertise

in mathematics curriculum, instruction, and assessment that cover the entire spectrum of school mathematics, K-12. Each evaluator has extensive knowledge of the Florida mathematics standards and best practices for teaching and learning mathematics. Two of the reviewers worked on the development of the new Florida mathematics standards – one as a framer and one as a writer (see <http://www.flstandards.org>). One of the evaluators is also serving as a consultant related to the development of the FCAT assessment that will measure student achievement of the new standards. In sum, each evaluator has a strong record of involvement in mathematics education at the local, state, and national levels.

### **Content Analysis of the SDHC Elementary Mathematics Program**

This evaluation report reflects the results obtained from the analysis of the School District of Hillsborough County (SDHC) Elementary Program, *Voyages*, as presented by the materials selected to implement the program. Specifically, we were asked to do the following:

- a content analysis of *Voyages* and other mathematics materials based on the current standards (Florida Department of Mathematics, 1996), and
- an appraisal of the same resource materials in relation to the new standards (to be adopted by the Florida Board of Education in 2007). [Available at <http://www.flstandards.org>]

To conduct the review, each of the evaluators was provided a set of the *Voyages* curriculum materials and provided access to the database of lessons that are available to teachers (<http://www.sdhc.k12.fl.us/instruction/curriculum/>). The components of *Voyages* that were reviewed include the Kindergarten Kit (e.g. teacher’s edition, student activity book, Island and Excursions Card (set A, B, & C)) and the following materials for each grade level 1 – 5:

- Teacher Edition *Excursions*
- Teacher Edition *Anchors*
- Student Edition *Excursions*
- Student Edition *Anchors*
- Topic Assessment and Teachers Guide (Form A and B)
- Subtopic Assessments
- Teachers Guide subtopic assessment

Conclusions drawn as part of this evaluation are based on the examination of these curriculum materials and of additional materials submitted to the evaluators by the publisher. The finding provided in this report is not supported or verified by an analysis of the enacted or achieved curriculum or the professional development program offered by the school district. As such, this evaluation represents one aspect of curriculum implementation.

### **Evaluation & Analysis Questions**

The examination of the SDHC Elementary program, *Voyages*, was guided by the following evaluation and analysis questions:

1. To what extent does *Voyages* support its intended framework and vision (e.g., programmatic theory) and stated objectives? To what extent is support information provided to permit the curriculum to be delivered with fidelity of implementation?

2. To what extent does *Voyages* address the intended curriculum as outlined by the 1996 Florida Sunshine State Standards- Mathematics?
3. To what extent does *Voyages* address the new Florida mathematics standards to be adopted in 2007?

To address Question 1, each evaluator reviewed the information that describes the nature and intent of the *Voyages* and examined the extent to which stated goals and objectives are supported throughout the curriculum. In completing this stage of the review, the intent of the evaluators was to understand the nature and intent of the curriculum as described by the designers' goals so that we could acknowledge any biases that might influence the results of the evaluation. As we engaged in the review, we continually examined whether the actual materials were consistent with described vision and goals for the curriculum. This type of analysis and reflection occurred throughout all stages of our review.

We examined the information provided to teachers to assess the extent to which teachers could use this information to support their implementation of the curriculum. Prior to discussing our assessment, we believe it is important to acknowledge that highly qualified and skilled teachers would be able to make professional judgments about the information provided based on their professional knowledge obtained as part of their pre-service or in-service education. That is, many of the comments made below may appear to be obvious to skilled and knowledgeable teachers. However as we conducted this review, we did not focus on the practices of teachers who would be considered highly qualified and effective. Such teachers are likely to be successful regardless of the curriculum that is used. Based on our experiences and what is known about teachers' mathematics content and pedagogical content knowledge (Fennema & Franke, 1992; Ball, D. L., Lubienski, S. & Mewborn, D. S., 2001), teachers who are challenged to implement the curriculum, particularly innovative curriculum that appears to be different from the norm, tend to be those teachers with insufficient knowledge to determine best approaches to use or to support students' mathematical development. As a result, these teachers are more likely to rely specifically on information made available as part of the textbook. Because of this, we examined the *Voyages* curriculum materials to assess whether sufficient information is provided to allow an average teacher to make sense of the content development and the intended instructional practices. That is, what could a teacher glean from the curriculum materials if opportunities for professional development were not available or were not attended by the teacher? Particular features of the curriculum and lessons were examined to determine whether they were appropriate and sufficient to support the implementation of the elementary mathematics program as intended. In particular, we examined the following:

- curriculum guidance information (e.g., descriptive information & pacing guides),
- development of the mathematics (organization of materials, clarity of explanation, examples, etc),
- lesson components (problems and exercises provided to support student learning, opportunities for review of prior content, homework, etc.), and
- development of basic, conceptual understandings, and procedural understandings.

These features were judged against "best practices" for teaching and learning mathematics. The following are eleven important and interrelated characteristics of the best practices of teaching mathematics embodied in NCTM (1989, 1991, 1995, & 2001) documents, which supported and guided our content analysis:

- The goal of teaching mathematics is to help all students develop mathematical power (build understanding and meaningfulness).
- Teaching for mathematical power requires providing experiences that stimulate students' curiosity and build confidence in investigating, problem solving, and communication.
- How well students come to understand mathematics ideas is far more important than how many skills they acquire.
- Mathematics is not a set of isolated topics, but rather an integrated whole.
- Problem solving is the focus of a curriculum that fosters the development of mathematical power. In this context, problem solving means more than just “word” or “story” problems used in more traditional approaches.
- Students need many opportunities to use language to communicate mathematical ideas.
- Reasoning is fundamental to knowing and doing mathematics.
- Concepts of numbers, operations, and computation should be broadly defined, conceived, and applied.
- The concepts of geometry and measurement are best learned through experiences that involve experimentation and the discovery of relationships with concrete materials.
- The understanding of statistics, data, chance, and probability comes from real-world applications.
- The major purpose of assessment is to help teachers better understand what students know and make meaningful decisions about teaching and learning activities.

To determine the extent to which the curriculum addressed the Florida mathematics standards (Question 2), the evaluators first had to reach consensus regarding what is meant by “addressing” or “meeting” the standard or grade level expectation (GLE). The following criteria were established to guide our independent review:

The lesson or collection of lessons that addressed the standard and its corresponding GLE had to:

- provide students an opportunity to learn the content described by the standard and its corresponding GLE.
- reflect the type of learning encouraged by the mathematics community. Specifically, we determined whether the lesson reflected and addressed the process standards delineated by the National Council of Teachers of Mathematics (2001), *Principle and Standards for School Mathematics* and supported by the Florida Framework for Mathematics (e.g., problem solving, reasoning and proof, communication, connections, and representations).
- provide opportunities for students to develop both procedural and conceptual understanding.
- Expose students to the breadth of knowledge needed to make sense of the mathematics topic and extend their learning beyond the ideas presented by the lesson.

Using the above criteria as our standard, we engaged in a two-tiered analysis of the curriculum. First, we engaged in a fine grain analysis of the curriculum to determine whether the curriculum provided resources to teach the content of the intended curriculum. At this level of the analysis, we were attempting to determine whether lessons were available to teach the content of the standards and not the quality of the lesson. Second, we engaged in a qualitative analysis of the nature of the curriculum to determine whether the approaches used had the potential to support

students’ learning, incorporate recommended best practices, and permit teachers to implement the lessons as intended. These two approaches were necessary to obtain a comprehensive view of the elementary mathematics curriculum used by the school district.

As part of the fine grain analysis, each evaluator conducted a thorough review of four of the six grade levels materials (see Table 1). Each evaluator independently reviewed the content of the materials for the assigned grade level and recorded information on a chart, examining whether the strand, standard, and grade level expectation (GLE) are addressed in the curriculum. While engaged in this review, each evaluator documented reflections about the nature and quality of the lessons and other concerns that could be used in the qualitative review of the curriculum. Then, the results of the independent reviews were compared to determine level of agreement. In cases where there was not agreement, the third evaluator reviewed the curriculum and contributed to the deliberation until consensus was achieved. Finally, if a standard or GLE was not addressed by the *Voyages* curriculum, the elementary mathematics lessons database was examined to determine whether teachers had access to other lessons or activities that would address the particular standards or GLE.

Table 1: Voyages Grade Level Materials Reviewed by Each Evaluator

	K	1	2	3	4	5
Evaluator 1		X	X	X		X
Evaluator 2	X		X	X	X	
Evaluator 3	X	X			X	X

To answer Question 3, it was important to determine the extent to which the elementary mathematics curriculum could be used to deliver the Florida mathematics standards as proposed. To answer this question, it was important to consider the paradigm shift that is apparent in the structure of the curriculum and the pedagogical practices that are needed to support student learning as intended. To accomplish this task, we first summarized differences between the 1996 Florida mathematics framework and ones to be adopted in 2007. Then, we identified needed features of a curriculum that would support the implementation of the new curriculum.

In reporting our findings, we provide examples from the curriculum materials across a variety of grade levels. To be included as part of this report, the conclusions had to be made across multiple grade levels.

### **Voyages: Developer’s Vision for Implementation**

This section describes the intent of the *Voyages* curriculum as described by the developers. This information was gathered from the introductory materials that are provided in each text under the following headings: *Welcome to the Voyages Mathematics Program*, *More about Voyages*, *Supporting and Developing the Language of Mathematics*, and *Addressing the Needs of All Students*. Additional information about the intent of the program was obtained from a document provided by the publisher that is entitled, “*Voyages Mathematics Program: Research Base*” (See Appendix A)

## *Types of Lessons*

The Voyages curricula materials are composed of two types of lessons: Excursions and Anchors. According to Cambium Learning (December 2006),

Excursions lessons are teacher-led, multi-day, activity-based, interactive lessons that emphasize the development of students' conceptual understanding. Anchor lessons, when used in conjunction with Excursions lesson, provide embedded enrichment, offering students the opportunity to independently use the knowledge developed during the Excursions lessons through a step-by-step process that provides context and opportunity to develop computational fluency. (p. 7)

Excursion lessons are intended to build upon students' knowledge by providing students opportunities to solve problems. Each lesson includes the following sections:

- Setting the Stage – establishes the psychological set and accesses prior knowledge
- Building Conceptual Knowledge – presents related concepts and mathematical language
- Building Skills and Strategies – includes a set of skills and strategies that may be used in application situations that often go beyond traditional approaches.
- Putting It into Action – shows and uses application of knowledge
- Making Connections – relates mathematics being learned to other disciplines and/or other mathematical topics

The curriculum is designed to provide opportunities for the development of both procedural and conceptual fluency. Students are expected to develop a variety of solution strategies and should be provided ample opportunities to reflect on their thinking with pictures, writing, and creative algorithms. Although not listed, each lesson also includes a lesson overview.

## *Pedagogical Approaches*

The type of instruction that supports the goals of the *Voyages* curriculum is one in which students are encouraged to engage in mathematical inquiry in a learner centered environment. While engaged in excursions lessons, students are expected to explore mathematics using prior learning experiences, make and test conjectures, present, explain, and justify solution strategies and question strategies presented by others. Teachers are expected to facilitate student learning by orchestrating the classroom discourse in ways that build students' development of mathematics language. Lessons involve learning experiences that include links between hands-on concrete representations to pictorial representation to more abstract learning experiences.

## *Assessments*

The assessments that are part of the *Voyages* curriculum are intended to be used by teachers to guide instructional decision making. "Assessment is also authentic: focused on student understanding, the assessment items require students to solve complex problems that are situated in real-world problem context" (Cambium Learning 2006, p. 8). Teachers make decisions about what assessment to use and provide instruction based on their professional discretion.

### *Meeting the Needs of All Students*

Although the materials state that the authors have “compelling evidence about how children learn mathematics” (p. viii), there is no clear indication of what that evidence is or about how that evidence was used in the development of the curriculum. Regarding meeting the needs of learners, it is stated that lessons are structured to address various learning modalities (kinesthetic, verbal/linguistic, auditory, visual/tactile, visual/spatial, & interpersonal) to meet the needs of all students.

### *Curriculum as Professional Development*

The developers posit that professional development is an embedded part of the curriculum. “Voyages is more than just a curriculum geared for student learning, it is also a professional develop[ment] program rooted in the research about how teachers benefit most from professional development and grounded in the daily use of curriculum in the classroom” (Cambium Learning, 2006, p. 11). According to the developers, teachers engage in individualized professional development in the mathematics classroom that is related to mathematics content, student thinking, and instructional strategies.

The use of curriculum materials such as those included with *Voyages* can provide teachers opportunities to explore their own mathematical knowledge as well as that of students at their own pace, and has the potential to provide teachers with a source of career-long, individualized, sustained opportunities for professional development because it is embedded within their daily work with students. (Cambium Learning, 2006, p. 13)

*Voyages’* built-in professional development program builds upon existing knowledge by asking teachers to model the process of inquiry along with their students during the use of Excursions lessons. (Cambium Learning, 2006, p. 13)

The intent is for teachers to learn and deepen their content knowledge while they teach students using the *Voyages* curriculum. Apparently, through the use of both types of lessons, teachers will learn about the standards and the “big mathematical ideas” supporting each instructional unit. The publisher indicates that teachers are not told what to do because such an approach would limit teacher development and learning.

*Voyages* focuses on developing an understanding of how children learn mathematics, while also providing built-in support that helps make connections for and with children and scaffolding instruction so that concepts build upon prior understanding and connect conceptual and procedural knowledge. (Cambium Learning, 2006, p. 14)

The *Voyages* curriculum is scaffolded to provide the teacher with instructional techniques to try at his or her own pace in his or her own classroom setting. Helping teachers transition toward Excursion-type instruction using Anchors lessons as a support is an effective, supportive way to ensure that students are still getting what they need as the teacher is encouraged to try new pedagogical strategies for enhanced student learning. (Cambium Learning, 2006, p. 15)

Overall, the *Voyages* is conceived as an innovate approach to enhance both teaching and learning.

## To What Extent Does *Voyages* Support Its Intended Framework and Vision (e.g., Programmatic Theory) and Stated Objectives?

We present our answer to this question by discussing the treatment of identified features of the curricular materials under several headings. First, we begin providing our evaluation of the materials provided as part of the text to support teachers' implementation of the curriculum. Then, we provided our evaluation of the curricula material potential to serve as individualized professional development for teachers as described in the document "*Voyages Mathematics Program Research Base*" that was provided by Cambium Learning. As mentioned earlier, we approached this analysis from the lens of a teacher who is using the materials without the benefit of additional information that may be provided as part of a professional development program.

### *Support and Interpretive Information Found in Voyages*

Common information is provided in each teacher's edition of the *Voyages* curriculum. It includes the following components:

- *Welcome to the Voyages Mathematics Program*
- *More About Voyages*
- *Supporting and Developing the Language of Mathematics*
- *Addressing the Needs of All Students*
- *Pacing Calendar for the Year*
- *Coverage of Grade Level Expectations for the Sunshine State Standards*

In this section of the report, we comment on the information provided in each of these sections. In particular, we highlight areas where clarification might be helpful. In doing so, we readily acknowledge that some of the suggestions or recommendations are not necessarily addressed in comparable curriculum materials. We purposefully decided not to limit our review with this in mind. We deemed it more appropriate to provide information that could alert the mathematics program to potential areas of difficulty and that may be used to enhance information provided to teachers. One might argue that teachers do not read this information. However, we contend that these resources could be used as a valuable resource to support teacher learning. That is, provided information ought to provide insights about the curriculum and instructional practices encouraged to support student learning.

#### *Welcome to the Voyages*

This section is intended to help the reader understand the intent of the curriculum. We found that several broad statements are made without clarification. This section begins with the following statement: "The **Voyages** program has been developed in response to identified and needed Expectations and Standards, and to best practices in mathematics education" (Bold in the original, p. vii). No clarification was provided for what Expectations and Standards were "identified" or "needed." The second sentence states "All lessons compact the curriculum and accelerate the instruction" (p. vii). However, no clarification is provided to describe or explain what is meant or to clarify how this was done within lessons. The third sentence, "Voyages is designed with on-grade level floors and it eliminates grade-level ceilings" (p. vii), does not provide information that clarifies what is meant by this statement. The remainder of the section describes the Excursion and Anchors lessons and provides a description of sections that are found within them. This section ends with the following statement: "Both types of lessons are

needed to develop mathematical proficiency”. (p. vii) This statement is not as strong as similar statements made by the developers where it is clearly indicated the Anchor lessons are intended to support and reinforce learning garnered as part of the Excursions lessons. That is, this statement might not help teachers understand that the intent is to engage students in problem solving and the development of conceptual understanding of mathematics, and language skills prior to engaging in direct instruction that presents particular strategies for approaching the mathematics. We found that the Excursions lessons provided the most opportunity for the type of instruction recommended for use by the mathematics community (NCTM, 2001). Although students can learn mathematics through the use of the Anchor lessons, they do not encourage the development of the breadth and depth of understanding that is expected by the Florida *Sunshine State Standards*. As a result, we wondered whether sufficient information was provided to help teachers understand the important role that the Excursion lessons play in development of students’ mathematical development. It is important to acknowledge that the *Topic Itinerary* does provide a sequence of lessons to be followed. Typically, Excursion lessons are followed by Anchor lessons related to the same topic. However, in not clearly delineating an approach for instruction early on, teachers might be led to minimize the significance of using both types of lessons. Teachers who opt to focus only on the Anchor lessons will limit the curriculum substantially.

### *More about Voyages*

This section provides additional information about the program. Below we identify some areas that we believe need some clarification. In particular, we considered the following statements made about the *Voyages* program:

- Provides strong and consistent development of the language of mathematics.
- Builds ties from arithmetic to mathematics, including algebra as generalized arithmetic.
- Provides strategies for teaching in a variety of modalities. (p. viii)

Regarding the first two items, clarification as to how this is done throughout the program should be provided. In particular, it should be clarified that the teacher plays a significant role in supporting students’ development of the language of mathematics by utilizing the resources provided in the curriculum, such as the glossary and attention to important vocabulary.

Regarding the second item, we questioned whether teachers would readily understand what is meant by “algebra as generalized arithmetic.” From a research perspective, this notion goes beyond the replacement of the missing value box to an unknown variable (Kieran, & Chalouh, 1993; Usiskin, 1988). Informed teachers would understand that the intent is to support students thinking in ways that encourage them to recognize patterns and make generalizations. The development of “algebra as generalized arithmetic” is not well-developed as part of the curriculum. For example, some of the information provided under the “Algebra Readiness” component in the teacher’s edition does not clarify how the identified statement encouraged the development of students’ algebraic thinking. The information provided is valuable, but elementary teachers might need professional development to help them fully understand their role in developing students’ algebraic reasoning skills. Many elementary teachers do not readily see the connection between the mathematics that they are teaching and its link to students’ algebraic development.

### *Supporting and Developing the Language of Mathematics*

A teacher's ability to support students' development of the language of mathematics is reliant on the teacher's ability to support students' use of mathematics language by incorporating various strategies to help students make meaning of the mathematics in all of its representations (oral, written, symbolic, or visual) (Thompson, Kersaint, Richards, Hunsader, & Rubenstein, In press). Some strategies are provided to help teachers support language development, but the approach used to present this information is limited. It does not reveal the need for teachers to continually support students' development nor does it help the teacher recognize the need to use a variety of approaches. Beyond the information provided in the teacher's edition, professional development would be needed to help teachers develop strategies to support students' language development. The following features are identified to support the development of the language of mathematics: highlighting of words, the glossary, and recycling of terms. These features are valuable only to the extent that teachers emphasize them and use them as part of their instruction.

### *Addressing the Needs of All Students*

Like in other sections, we believe that clarification is needed for statements made in this section in order for teachers to find the information useful. For example, a claim is made regarding "compelling evidence about how children learn mathematics," yet the source of this evidence is not provided. Was it based on personal experiences or research? Although there is research support for some of the claims that are made, no links to such research are made (see for example, National Research Council 2001). This section describes how a variety of modalities (e.g., kinesthetic, linguistic, auditory, tactile, spatial and interpersonal) are represented in the lessons and they were "conceived as a means of support for specific student populations," but the populations in question are not identified. Based on our professional judgment, we do not believe the provided descriptions support teachers in their understanding of the use or benefit of these different modalities. Again, this is an area where teachers would need further clarification and professional development.

### *Coverage of Sunshine State Standards Chart*

Without additional information, it was difficult to interpret the information provided in this chart that is provided as in the teacher's edition for every grade level. On its face, we understand that the intent of the chart is to identify lessons that address the particular standards, benchmark, and GLEs. A cursory glance might lead one to believe that most of the standards and benchmarks are addressed and that some are addressed thoroughly because they are addressed in multiple lessons. Our in-depth review of the materials support this conclusion in most cases, but also raises several issues related to the display of information on the chart. These issues are described below:

- Interpretive information is not provided to help teachers understand how to determine whether or not they have met the intended standard. On the chart multiple lessons are identified as evidence of meeting the standard. For example, in the Grade 5 teacher's edition, 20 lessons are identified for meeting Benchmark MA.A.3.2.3. How is a teacher to interpret this? Is it sufficient if only one lesson is taught or must all of the listed lessons be taught in order to ensure that this benchmark is adequately addressed?

- There are cases where multiple lessons are identified; however, only a few directly address the intended benchmark. For example, three anchor lessons (topic 2: Lessons 23 & 24 and Topic 6: Lesson 7) are listed to meet Grade 4 Benchmark MA.A.1.2.3 -- “translates problem situations into diagrams ...” however, Lesson 23 and 24 do not require students to address problem situations. Please note that problem situations are addressed in the Excursion lesson.
- In some cases, prerequisite lessons that represent a foundation to enhance student learning are also identified. Although the prerequisite lessons may be needed to develop students’ understanding, we questioned the listing of multiple lessons that do not clearly address the identified benchmark.
- In some cases, a benchmark addresses more than one outcome (e.g., explain and demonstrate) or multiple context (e.g., including decades, hours, minutes, and seconds) but the lessons may not address all of the intended components. Examples are provided from Grade 3 to illustrate how particular aspects of the standards are not addressed in identified lessons.
  - MA.A.1.2.1 (GLE 2) – Fraction denominators are not addressed through 100.
  - MA.A.1.2.1 (GLE 3) – Practice is not provided for reading decimal notation.
  - MA.A. 1.2.2 (GLE 2 & 3) – Concrete materials are not used.
  - MA.A.1.2.3 (GLE 1) – Decimals are not addressed.
  - MA.A.1.2.4 (GLE 1 & 3) – Percents are not addressed.
  - MA.A.4.2.1 (GLE1) – Does not address multiplication or division.
  - MA.A.4.2.1 (GLE 2) – Factors are not found through 100.
  - MA.B.1.2.1 (GLE 2) –Weight, capacity, area, & length are not directly addressed.
  - MA.B.1.2.3 (GLE 3) – There is no discussion of calendar.
  - MA.B.1.2.3 (GLE 2) – Perimeter is not addressed.
  - MA.B.2.2.1 (GLE 1) – Beyond volume the other measurement types are not addressed.
  - MA.B.2.2.1 (GLE 2) – Practice is not provided for comparing lengths that cannot be physically compared.
  - MA.C.3.2.1 (GLE 1) – Area and perimeter are not compared.

In each of the cases identified above, a skilled teacher could easily extend the lesson beyond the information provided in the text to ensure that the intended Standards and corresponding GLEs are fully addressed. As part of Excursions lesson, a teacher could ask questions to encourage and extend students’ thinking. In addition, other resources are available at the school district, such as the lesson plan data base, to allow teachers to address the intended standards. However, no specific guidance is provided in the text to support teachers in this area.

- Benchmarks that involve “Explanation” are primarily addressed as part of the Excursions Lessons. This is not problematic as the Excursions lessons have the potential to provide students ample opportunities to explain their thinking and solution strategies. However, two assumptions are made. One assumption is that the teacher will teach the Excursions lessons. Another is that teachers will facilitate the lessons in ways that encourage students to explain their thinking. Based on our experiences, we are concerned that this may not occur effectively in classes where teachers have insufficient mathematics background or lack an understanding of appropriate pedagogy to support student learning.

- Real-world problem solving is not addressed consistently as part of the curriculum materials. That is, some lessons that are identified to address problem solving are limited in the extent to which real world problems are addressed. In some cases, we question whether the identified lesson adequately addressed problem solving. In other cases, problem solving is not addressed as required by the identified benchmark. For example, the *Voyages* authors identified four Grade 4 Anchors Lessons (Lesson 8: Topic 2, Lesson 10: Topic 4, Lesson 27: Topic 4, and Lesson 14: Topic 6), and two Excursions lessons (We are family: Topic 4, and Frisky frog fractions: Topic 6) aligned with GLE MA.A.3.2.2.1 (uses problem-solving strategies to determine the operation(s) needed to solve one- and two-step problems involving addition, subtraction, multiplication, and division of whole numbers, and addition and subtraction of decimals and fractions). This combination of lessons met the first part involving operations with whole numbers, but did not meet the second part of the GLE involving addition and subtraction of decimals. In another example, the authors identified one Grade 4 Anchors lesson (Lesson 10: Topic 2), and one Excursions lesson (Group Rate: Topic 4) as aligned with GLE MA.A.3.2.3.1 (solves real-world problems involving addition, subtraction, multiplication, and division of whole numbers, and addition and subtraction of decimals and fractions using an appropriate method; for example, mental math, pencil and paper, calculator). This combination of lessons met the addition, subtraction, and multiplication part, but did not meet the division operation component of the GLE.
- At times the lesson identified does not adequately support the intended standard. An example of this is illustrated in the Grade 4 materials (MA.A.3.2.1, GLE 6). The Excursions lesson focuses primarily on the distributive property of multiplication over addition. However, other properties are intended to be addressed as part of this benchmark (e.g., zero identity, commutative, and associative properties). Although the other properties could be reviewed by the teacher while implementing this lesson, there is no guidance to include a focus on the other properties. Again, meeting this standard is dependent on teacher knowledge and facility with instructional approaches that encourage students to connect mathematical ideas.
- Some lessons are tangentially related to the intended standards. For example, students might be expected to count during a lesson on probability, but it might not be appropriate to use that lesson as a basis for determining that students have met the standard that is directly related to counting. The connections between the standards might be weak if the intent is not made explicit. In addition, we question if it is appropriate to label a lesson “probability” simply because students are using a spinner to count results.

Overall, given the need to provide additional interpretative information and to clarify intent, we believe teachers will need assistance and professional development in order to understand the intent of some of the messages provided as part of the text.

## Curriculum as Professional Development

Although the curriculum was designed with the intent to provide individualized professional development for teachers, we have determined that the *Voyages* curriculum falls short in this regard. As reported earlier the publisher's report that the curriculum is intended to serve as individualized professional development to strengthen the teacher's understanding of mathematics content, student thinking, and instructional strategies. However, many of the lessons do not provide the sufficient information to support teacher learning. As described, the Excursions lessons are intended to be interactive lessons that allow teachers to guide students to deeper understanding of mathematics. The descriptions provided in the teachers' edition for the excursion lessons do not provide teachers with the depth of knowledge needed to facilitate student learning. There appears to be the assumption that the teacher can take the information and develop a well-structured lesson that will enhance students' understanding. Given what is known about elementary teachers' conceptual understanding of mathematics (Fennema & Franke, 1992), it is clear that additional information needs to be provided to support teachers with insufficient mathematics background or experience.

Teachers are guided in the phases of the lesson, and provided information that outlines what they are expected to do and what knowledge students are intended to develop. An already knowledgeable teacher would be able to implement the lesson and elicit the type of knowledge expected from students. However, a teacher with insufficient mathematics or pedagogical background would need additional information to support the effective implementation of lessons. In particular, additional clarification is needed about the intent of the lesson to help teachers determine appropriate approaches to develop students' conceptual understanding of mathematics. Overall, the lesson descriptions emphasize things that teachers ought to do during the lesson (teacher-centered), but do not always provide a sufficient rationale that supports why it should be done. If rationales were provided, teachers would not only know what to do, but they would also understand why engaging in suggested actions is expected to lead students to greater mathematical understandings and this would allow the teacher to understand the nature of appropriate instructional approaches to support student learning (i.e., student-centered activities). Further, information needs to be provided that would support teachers in making connections, providing extensions, addressing potential areas of student difficulties, and that would sustain teacher mathematical and pedagogical growth. Several illustrative examples are provided below to illustrate and clarify the comments made about the use of the curriculum as professional development.

### *Grade 4: Topic 6 – “To Round or Not to Round”*

In the section entitled, “Building Conceptual Knowledge”, teachers are guided to help students understand the size of fractions by relating them to benchmarks of 0 and 1, and then later to half as a means to help students determine how to round. Although the overall approach is appropriate and could potentially lead to students' understanding, it is not clear whether a teacher with limited content knowledge or who is limited in pedagogical approaches would be supported in their efforts to implement this lesson. To develop teachers' content and pedagogical knowledge, there is a need to help teachers understand what mathematics understandings are intended to be developed. It is never explicitly stated that the intent of the lesson is to help students use benchmarks as a means of determining the size of a fraction. The lesson overview states the following: “Students are introduced to the concept of fractions' being less than, equal

to, or greater than one half.” The intent of the word “concept” is not clear as used here. The concept to be understood in this case is the “relative size of fractions”. The means by which this concept is being developed in this lesson is to compare them to commonly used benchmarks. It is important that the curriculum clearly identified the concept to be developed so that teachers can ascertain whether they are in fact developing students’ understanding.

Information is not provided with the description to help teachers understand the nature of the learning environment that is intended. For example, this lesson states “Tell students that when a fraction’s numerator is much smaller than the denominator, the fraction is closer to 0 than it is to 1.” Statements such as this do not support the vision for inquiry described for use with the curriculum. Students could be led to this conclusion by exploring the relationship between the numerator and denominator of various fractions and drawing their own conclusions about the relative size of the fraction. Teachers could be guided in the types of questions to ask, the expected student answers, and follow-up and summary statements to ensure accurate mathematics understandings. The same could be done in relation to the benchmark of  $\frac{1}{2}$ . Finally, students could revisit their prior understanding of what it means to round and formulate a conclusion related to how to round with fractions. The recommendations provided above would 1) help teachers understand the mathematics understanding they are intended to foster, and 2) provide opportunities for students to build on their prior understandings to make connections to what they are expected to learn as part of this lesson. To further support teacher development, there is a need to clarify what it meant by “Work through the problems with students” that is provided in the “Building Skills and Strategies” sections particularly as it relates to supporting student inquiry. Given the vision for the use of the curriculum, one can surmise that the expectation is that students share their responses, provide a rationale for their response and be given opportunity to question each other’s reasoning until they are moved to greater levels of understanding. However, the instruction provided to teachers does not clarify expectations regarding the teacher’s or the students’ role as they engage in this segment of the lesson.

To help teachers meet the needs of a diverse student population, teachers need to know about alternate approaches that could be used to foster students’ knowledge development. For example, in addition to the area model, illustrated by circles, the same concept could have been explored using a linear model (e.g., Cuisenaire rods or number lines). The use of multiple visual representations would provide students various access points to understanding.

#### *Grade 2: Topic 4 – Mile by Mile*

In this example, it was unclear whether the intent was to explore approaches that students would use to solve the problem or to simply provide students a predefined approach for solving the problem. Again, focusing on the vision outlines for the curriculum, one would assume that the goal would be to explore students’ strategies as supported by research. To support teacher development, it might have been helpful if teachers recognized that there is a significant amount of research that supports the use of *Cognitively Guided Instruction* (Carpenter, Fennema, Franke, Levi, & Empson 1999). This research suggests that children can develop their own strategies for approaching a wide range of problem solving situations. In particular, when students are asked how they would approach a problem, they are able to generate appropriate approaches to obtain a solution. This would help a teacher understand that he or she need not be the sole source of problem solving strategies. Instead, students can be led to describe their approaches for finding solutions. The lessons as described do not provide teachers any background knowledge to support the use of any particular strategy. Teachers are guided in various components of the

lesson (e.g., “Have students ...”, “Tell children ...”). In addition, teachers are told to “Discuss with children what happened to the digit in the tens place when the digit in the ones place became larger than nine.” To support the nature of the classroom discourse as intended, it might have been helpful if teachers were provided information to clarify the intended mathematics. That is, information needs to be provided to ensure that teachers understand the mathematics content they are expected to discuss with students, possible student misconceptions, and approaches to foster student learning and meet the needs of a diverse group of students. Classroom discussions can be used to encourage students to develop greater understanding (see for example, Chapin, O’Connor, & Anderson, 2003). Also, the section “Explore modeling regrouping” suggests that teachers should model two approaches that are presented. Rather than the focus being on the teacher (i.e., teacher-centered activities), an alternative approach would be asked students to solve the problem and share their solutions strategies (i.e., learner-centered). As part of this process, the teacher could facilitate a discussion about the different approaches presented by students highlighting similarities and differences. The methods could have been presented as possible approaches that are available for use rather than particular approaches to be modeled. Again, professional development that focuses on students’ thinking (e.g., Carpenter et al., 1999) could provide teachers the information needed to support student learning.

The examples presented above illustrate difficulties associated with using the *Voyages* curriculum as a source of individualized professional development as intended by the developers. To fully facilitate teacher learning, the teachers’ edition would need to provide teachers explicit information that

- clarifies the intended instructional approaches to be used,
- supports teacher use of instructional strategies,
- recognizes that teachers may not have the conceptual knowledge of the topic needed to support student learning, thus clarifying the intended mathematics understanding with examples, and
- provides teachers with approaches they can use to meet the needs of a diverse student population by providing alternative approaches that are available to meet the needs of all learners.

In addition, other resources should be identified that teachers can use to further their understanding of both content and pedagogy.

### **To What Extent does *Voyages* Address the Intended Curriculum as Outlined by the 1996 Florida *Sunshine State Standards- Mathematics*?**

#### ***Lessons Available to Address the 1996 Standards***

To answer this question, we examined the primary elementary curriculum, *Voyages*, and the lesson database using the fine grain analysis to determine whether teachers had access to lessons that would cover the content of the SSS curriculum. Based on our analysis, we conclude that lessons exist within the SDHC elementary mathematics program to address the mathematics content addressed in the 1996 Florida mathematics standards. With the exception of Standards that address technology, during our analysis we were able to identify a lesson or lessons that address each and every Standard, Benchmark and GLE. So if the overall concern about the curriculum is the availability of lessons, then we have determined there is no issue in this area.

*“...lessons exist ... to address all the mathematics content addressed in the 1996 Florida mathematics standards.”*

However, according to NCTM (2001) “a curriculum is more than a collection of activities [or lessons], it must be coherent, focused on important mathematics and well-articulated across the grades” ( p. 14). A curriculum that is coherent links and builds on mathematics ideas to lead to depth of understanding. “A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills.” (NCTM 2001, p. 15)

A curriculum should focus on important mathematics including foundational ideas such as place value, equivalence, proportionality, function, rate of change and build profound understanding of these fundamental ideas. Additionally, the curriculum should allow students to “connect ideas across different areas of mathematics” (p. 15). “The curriculum should also emphasize the mathematics processes [e.g., reasoning & proof] that support the quantitative literacy of students” (p. 16). In addition, ideas within mathematics should be integrated in ways that allow students to connect ideas across different areas of mathematics. “A school mathematics curriculum should provide a road map that helps teachers guide students to increasing levels of sophistication and depths of knowledge” (p. 16).

Although we have determined that there are lessons available to address each topic in the 1996 Florida mathematics standards, we discuss several issues that are important to the implementation of the curriculum. We organize our discussion of the quality of the lessons found the *Voyages* curriculum by addressing recommended approaches for teaching and learning mathematics as determined by the mathematics education community as represented by the NCTM’s (2001) *Principles and Standards for School Mathematics*. Specifically, we address our evaluation under the following headings: Communication; Problem Solving, Reasoning, & Proof; Connections, Representations, and Technology. Examples are drawn from the various grade levels to support conclusions that were drawn.

### ***Mathematical Communication & Language Development***

It is natural that communication be a concept that emerges from any discussion related to teaching and learning. Communication, oral, visual or otherwise, is the vehicle by which teachers present concepts and engage students in the act of learning. Through communication, teachers are also able to ascertain to what level students have understanding and knowledge about mathematics. Observe any active classroom and one will observe teacher-to-student, student-to-teacher, and student-to-student communication happening in an intertwined fashion. Communication involves many different activities (e.g., speaking, listening, writing, demonstrating, & reading) that are enacted for purposes of explaining, justifying, describing, debating, demonstrating, and the like. NCTM (2001) presents communication as a process standard to indicate that in the course of teaching and learning mathematics, teachers are to facilitate communication that actively engages students in the learning process. Furthermore, the Council posits that students’ learning is enhanced when they have opportunities to exercise mathematical knowledge through various means of communication with and about mathematics.

The *Voyages* curriculum is described as being positioned to support communication in the mathematics classroom. To narrow the focus of review, the evaluation team set out to determine if the curriculum supported classroom discourse, that is, academically focused dialogue between the teacher and students and among the students. The curriculum certainly presents a focus on

mathematics vocabulary that students should come to understand as a result of experiencing the curriculum. A list of terms is highlighted to showcase particular terms that should be defined within the context of instruction. However, an examination of how the lists of terms are addressed in the curriculum reveals little attention to the conceptual meaning of the terms or specific opportunities students should have to use or apply the terms. There are too few spaces in the curriculum where the teacher is guided in how to engage students in defining or applying the terms. Because there is not a strong emphasis on writing as a form of communication in the curriculum, teachers also miss opportunities for students to write their understandings as well as misunderstandings of key mathematical terms.

Throughout the list of GLEs where the expectation is for students to explain or justify, an examination of the curriculum reveals that either the expectation is superficial or overlooked altogether in the curriculum. There are few instances where the teacher is provided in-depth and probing questions that might forge such type of communication. The teacher ill-equipped to engage students in higher order thinking through the appropriate questioning will struggle with ways to support students' development of skills to explain and justify their responses either orally or in writing. Though communication appears to be embedded as part of the Excursion lessons, these embedded instances do not provide guidance to support teachers in orchestrating classroom discourse. This is particularly worrisome for novice teachers or teachers who are not adept as being facilitators of students' learning.

There are instances where it is clearly evident that the curriculum warrants communication. Prompts such as "make sure students..." and "discuss with students..." invite the teacher to engage the students in discourse. However, the curriculum does not help teachers determine ways to address discourse issues in the classroom. For example, just how is the teacher to "make sure students" know or can do something? Should the teacher expect the students to say or do specific things? Are there specific responses the teacher should expect from students? What is a teacher to do in the event the teacher is not sure that he or she has made sure that students can do or know something?

When it comes to discussion, the curriculum is weak in regard to preparing the teacher to lead discussion and in preparing students for what it means to discuss mathematics. There are a variety of discussion types (e.g., debates, sharing, convincing, etc.). Will any kind of discussion suffice at any point in the curriculum? When the teacher is discussing with students, is the discussion to be teacher-led or student-led? How much is the teacher to tell during these discussions? The curriculum provides little support for anchoring the inexperienced teacher in the notion of discussion and perhaps does not provide the support necessary even for the experienced teacher of mathematics.

When considering the notion of communication, one has to address the question of the format of the communication. The curriculum is projected to support communication in the classroom, but under what framework does this happen? The evaluators considered this question and searched for evidence of communication frameworks in the curriculum, frameworks such as student presentations, oral presentations, demonstration, alternative assessment, cooperative or group learning, team work, assessment of students' ability to communicate mathematically, etc. Essentially, the aim was to see how the curriculum provided a foundation to overcome unfocused talk in the classroom.

Essentially, the evaluation of the curriculum in regard to communication yielded the result that instruction emerging from the curriculum was very teacher-centered with limited opportunities for students to engage in planned and meaningful conversations in the classroom. Furthermore, Anchors and Excursions lessons fail to take advantage of opportunities for students to be engaged in peer-to-peer learning opportunities and to reflect on their learning. Assessment items that require students to communicate their mathematical ideas are also limited. For example, the majority of items require a response of a single numeral or a single word. There are few items that require students to explain, justify, or otherwise write about their understanding of mathematics. Whether or not a teacher can follow up this type of items with questions that promote in-depth communication is questionable.

### ***Problem Solving, Reasoning, & Proof***

The concept of problem solving was highlighted with the 1980 National Council of Teachers of Mathematics (NCTM) yearbook in 1980. At that time, a premier work in problem solving belonged to George Polya (1957). His steps of problem solving engaged the problem solver in such activities as making sure to understand the problem and looking back at the problem after obtaining a response to the problem. Problem solving continues to be a focus in mathematics education as evidenced by the presence of the concept as a process in NCTM's (2001) *Principles and Standards for School Mathematics* and in the myriad of publications that are available to support the teaching and learning of mathematics.

There are many issues to be concerned about relevant to problem solving. For example, what many consider to be problems are only exercises or applications of procedures. A problem is defined as a situation for which a response is required but the response as well as the route to the response is unknown. In addition, there may be only one layer or multiple layers to the problem, requiring the problem solver to pursue a collection of routes to the response. And furthermore, the response may be well-defined, a single response, open ended or even made of multiple responses. However, an exercise is classified as such because it does not challenge the learner to pursue a response, but rather provides practice for typical responses or simply gives the learner an opportunity to experience similar mathematics contexts in repetition. While exercises are important to the learning of mathematics, it is genuine problem solving that stretches the learner mathematically. It is in this vein that we examined the nature of problem solving in the *Voyages* curriculum.

Consider this example from Grade 4 Anchors (p. 329): "Four people paid \$320 to take a helicopter ride. What was the cost per person?" This is clearly an example of an exercise for Grade 4 students and it is reflective of the type of "problems" that are presented throughout the curriculum. However, there is documentation that the curriculum aims to hold true to problem solving as evidenced by lessons focused on "Multi-Step Problems" (Grade 4 Anchors, p. 316). Overwhelmingly, Anchors problems are of the routine nature – one-single answer type problems with generalized contexts.

Another side of problem solving is problem posing, particularly the posing of student-generated problems. The *Voyages* curriculum is essentially silent on this issue, losing an opportunity for students to connect with the mathematics via their real-life and relevant contexts. Along with problem posing are also opportunities for students to extend, modify, discuss and write about problems. These kinds of mathematical activities are not fostered by the design of the problem solving sections and items throughout the curriculum.

Problem solving steps or heuristics are useful guidelines for teaching students how to become effective problem solvers. The curriculum does not focus on the students being independent users of problem solving strategies – either those shared by the teacher or those generated by the students themselves. In many instances, the teacher is directed to tell the students how to solve the “problems”. Consider for example, Grade 4 Anchors (p. 466) where the teacher is instructed to “tell the class that these problems require them to find two distances, then to compare them to find which is farther”. Once the teacher tells the students this information, the core experience of the problem is erased as the students do not have to think through any problem solving process at all and essentially the problems are transferred into exercises for the students.

Underlying the problem solving process is students’ ability to work through contexts of problems. This is particularly important as students are being challenged to solve problems with real-life contexts. The *Voyages* curriculum does provide varied contexts for the problem solving sections. There are a few sections where contexts for problems almost disappear. Consider for example, Grade 2 Anchors lessons on geometry/measurement: 8, 9, 10, 11, 12, 14, 15, and 16. For the most part, the problem solving sections of these lessons focus on abstract concepts of geometry and/or measurement excluding attention to real-life contexts for such mathematical concepts as volume, congruence, and symmetry. In support of this position of the curriculum, students do need to be able to operate in the abstract in the work of problem solving.

It is quite common for a person to proudly say, “I’ve never been good at mathematics” and unfortunately these people come from every imaginable category, including that of school-age children. One impetus for this negativity is the perspective that mathematics is all about “crunching numbers” and if a person can’t crunch numbers then they often end up with the position that they are not good at mathematics. However, there are other facets to being engaged in mathematics such as the reasoning and proof.

To reason means to give mental energy to thinking about and through mathematics. It means to be engaged in asking questions about mathematics that lead to further understanding. It means challenging mathematical ideas, developing and testing hypotheses, questioning procedures, considering alternative routes and responses to problems, and reflecting on processes. Since reasoning cannot be seen, it can only be inferred from students’ responses (e.g., written, oral, demonstrative) to mathematics instruction with the success of reasoning being supported by students’ ability to be engaged mathematically (e.g., discussion, application).

Hence, students at all levels need opportunities to reason mathematically and these opportunities present themselves through the curriculum used for instruction. Along these lines, the evaluators considered the *Voyages* curriculum. In doing so, the result is that the curriculum does not adequately present opportunities for students to reason. For example, an overwhelming number of the mathematical exercises and problems for students to work through are void of instances where students have purposes for debating or questioning the mathematics. Almost all of the exercises and problems are single-response items, leaving very little room for students to discuss open-ended responses. Furthermore, there are too few opportunities in the curriculum for mathematics to emerge from the students, giving students room to use their own contexts to form mathematical thoughts. Reasoning skills are joined by such skills as explaining, justifying, describing, debating and proving – yet these are activities that are often over-looked by the curriculum.

In regard to reasoning and proof, students in grades pre-K-5 should have opportunities to engage in informal proof of mathematical ideas. All too often, students are taught to follow along mathematical procedures with little understanding of why procedures work or to use formulas with little understanding of what the formulas mean. These and many other areas build misconceptions for students and in later mathematical experiences, students are ill-equipped to think through mathematics if they can't remember a procedure or a formula. Proof can begin easily with students justifying their responses and grow as students engage in demonstrating specific and general mathematical notions.

Here is a simple example in the form of a question: Why does a negative number multiplied by a negative number result in a positive number? Now this is certainly a topic of study in university-level mathematics in the area of Abstract Algebra, but given that it is a notion that is presented in school mathematics, it is worthy of consideration at that level. Students can engage in informal proving of this mathematical concept for more solid mathematical understanding. It is an undertaking that will engage students in reasoning and proof with mathematics that is accessible to them – concepts such as inverse of number and the distributive property. The *Voyages* curriculum does not provide ample opportunities for students to be engaged in the learning of mathematics from this perspective.

### *Connections*

According to (NCTM, 2001),

[w]hen students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics. Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. Viewing mathematics as a whole highlights the need for studying and thinking about the connections within the discipline, as reflected both within the curriculum of a particular grade and between grade levels. (p. 63)

The *Voyages*' authors made a great effort to align specific lessons and activities to specific GLEs for each grade level. The alignment process and level of specificity are overall very exhaustive and complete. However, the interconnected nature of skills, concepts and topics could sometimes be diminished, or get lost at the end of such an exhaustive process. In other words, a lesson or activity could be very well aligned with a specific GLE, but it might not be properly interconnected with other mathematics concepts and skills in an appropriate context.

The *Voyages* lessons do provide some level of connection between mathematical concepts and skills. These connections tend to be more linear and teacher-directed in nature. The authors provided a spiral approach to the *Voyages*' curriculum by offering opportunities to practice previously covered content in the Connections section of the lessons. However, the exercises provided for this spiral approach do not necessarily lend themselves to in depth connections of mathematics concepts and skills. For example, the Connections section for Grade 5 Anchors Topic 3 Lesson 1 (p. 100) deals with measuring, identifying and classifying angles. It involves a

set of exercises asking students to use a given circle graph (divided in four sections for Animals at the Zoo: 25% Birds, 10% Reptiles, 20% Amphibians, and 45% Mammals), and a protractor to answer six questions: What percent of the animals at the zoo are amphibians?, What percent of the animals at the zoo are mammals?, Birds and reptiles represent what percent of animals?, What is the measure of the angle formed by the birds' section of the graph?, What is the measure of the angle formed by the reptiles' section of the graph?, and What is the sum of the angles for all four sections of the graph? As indicated by the authors, these "problems" prompted students to find information and data in a circle graph, use a protractor to find angle measures, find missing angle measures, and match terms in definitions. This activity provides practice exercises for previously learned ideas, and a certain level of connection between mathematics concepts (angle, percent, and circle graph) and skills (find information and data in a circle, graph, use a protractor to find angle measures and missing angles, and match terms with definitions). The Anchors Connections sections seem to follow this format for the rest of the series. It could be argued that the breadth and depth of the mathematics connections provided by these sections could be made more interactive by using other types of problem solving or application activities that allow students to explore ideas and connect ideas more effectively.

As indicated by NCTM (2001), it is very important not to lose sight of this interrelatedness of mathematics concepts and skills. The Excursions lessons tend to allow for this type of interconnected format in most cases. For example, Grade 5 Excursions Topic 1 Sabrina's Survey (p. 3) introduces students to a situation in which a girl plans how to help her father decide which video system to purchase for his toy store. The students will discuss ways to conduct a survey, and then organize given data in a circle graph, writing fractions as percents. Students also choose their own subjects for surveys, conduct the surveys, and show the results in circles graphs. This lesson provides a richer and more appropriate possibility for the interconnection of mathematics concepts and skills.

These types of connection of mathematics ideas that students need are very dependent on teachers' ability to orchestrate such a dynamic process in the classroom.

To emphasize the connections, teachers must know the needs of their students as well as the mathematics that the students studied in the preceding grades and what they will study in the following grades. As the Learning Principle emphasizes, understanding involves making connections. Teachers should build on students' previous experiences and not repeat what students have already done. This approach requires students to be responsible for what they have learned and for using that knowledge to understand and make sense of new ideas, (NCTM, 2001, p. 63).

The activities alone will not provide for the interrelatedness of mathematical ideas.

## *Representation*

Mathematical representations include the various concrete materials (e.g., pattern blocks, counters), pictures or diagrams, graphical displays, tables, charts, symbols or words used to model mathematical problems and situations. Using such representations is essential if students are to be given opportunities to develop conceptual understanding of mathematics. They are particularly useful for helping students visualize mathematics concepts and make sense of mathematical ideas. When developing conceptual knowledge, teachers should assist students as they move from concrete to pictorial to more symbolic representations. Translating among and between these representations allows students to transition to more advanced mathematical thinking.

When introduced to a new concept, students should be first provided opportunities to explore ideas using concrete tools. During this exploration period, students should be provided opportunities to make and share observations, justify or validate their thinking by testing out their ideas, and question or provide alternatives to conclusions drawn by others. These experiences permit students to recognize under what circumstance their ideas do or do not work, and what circumstances influence the mathematics being studied. During this time, students are engaged in critical thinking about the phenomenon that they are observing (e.g., “If I put these together, I get a ...”). Teachers should push students by asking questions to move their thinking forward (e.g., “Why do you think that happens?” “What do you think will happen when ...?” “Why don’t you try that out and tell me what happened?”). The use of appropriate questions can lead students to draw more robust conclusions about their observations. Once students investigate mathematics with concrete tools, they are better prepared to interpret and use pictorial representations of the hands-on models that they have used. They can connect the pictures to prior experiences and make judgments about their intended use as pictures. Finally, teachers should help students connect concrete and pictorial representation to their respective symbolic representations (e.g., “When we regroup here, it is like grouping the longs from the base ten blocks”). The same process from concrete to pictorial to symbolic can be used whenever a new concept is being taught. The use of multiple representations allows students to solidify their understanding and provides a variety of access points to understanding mathematics.

If used appropriately and creatively, the combination of Excursions and Anchors lessons are poised to provide students ample opportunities to explore mathematics ideas using a variety of representations at every grade level. In particular, a skilled and knowledgeable teacher could implement the lessons in ways that encourage students to explore mathematics, make and test conjectures, and support student learning as they consider the use of multiple representations. However, a teacher with insufficient or limited mathematics or pedagogical background may be challenged to use recommended representations or may use the representation in ways that do not lead to enhanced student learning experiences. For example, some teachers may struggle with the amount of time needed to engage students in mathematics exploration using a variety of representations. We each have encountered teachers in Florida who limit students’ mathematical experiences because they believe that they do not have time to spend on mathematics exploration prior to FCAT. These teachers do not recognize that the time spent helping students visualize and make sense of mathematics using multiple representations has a greater value in students’ long term mathematics development and results in prolonged understanding that is not provided when students are simply told what to do. In addition, students who develop confidence in their ability to process mathematical ideas individually and with peers are better able to address and solve a

variety of problems that on the surface may appear structurally unfamiliar or in a novel format thereby increasing the likelihood that students will attempt the challenging mathematics problems provided on FCAT. Overall, the extent to which students benefit from the use of a variety of representations is dependent on the ways that teachers facilitate students' learning with their use.

Any concerns that were raised about the use of representations in the *Voyages* curriculum during our evaluation more often than not focused on the directions provided for their use. As mentioned previously, rather than telling teachers to allow students to explore mathematics with the representations and share observations and findings, teachers are often provided "a step-by-step script" to be followed that tells them what to say and illustrates how they should direct students. We believe this is an attempt by the developers to provide guidance to teachers who may not be adept at orchestrating these types of lessons. At one level, they provide an overview of what should occur, however the instructions do not help teachers understand limits inherent in certain representations, misconceptions that may occur when used, nor appropriate methods to guide students to enhanced understanding. One might question whether this is the role of the curriculum materials or whether teachers should be assumed to have this knowledge. Unfortunately, available research and experience tells us that elementary teachers are challenged by mathematics topics that they are expected to teach and are challenged by a lack of pedagogical content knowledge (i.e., methods to deliver mathematics instruction in ways that are understandable to children). As we examined the materials we identified areas of potential problems related to the use of various representations. Below we provide examples to illustrate some of the discussion that ensued about the appropriate uses of mathematical representations.

#### *Grade 1: Topic 5 Measurement – Measurement*

*Measurement* is the first lesson recommended for introducing the unit on measurement. This lesson is to be followed by several Anchor lessons that involve measuring with standard and nonstandard units. This lesson is intended to address several standards that address measuring with standard and non-standard units. The passage below is provided for "Setting the Stage."

Tell children that the other day you wanted to measure a one-foot piece of string. Explain, *I knew that 12 inches is the same as 1 foot. I know that the last section of my pointer is about an inch long. So I used that part of my finger to measure the string.* Model measuring the string in this manner on the overhead. Explain that you used your finger as a referent for one inch. Then ask children how you check your estimate. Children should suggest that you could use a ruler. Use a clear ruler on the overhead to show that the piece of string is about 12 inches."

To help students understand the need to measure with standard units, students could be provided opportunities to measure items using a variety of objects (e.g., Cuisenaire rods, pencils, etc.) that represent nonstandard units and through this exploration students could be led to determine that in order to discuss the lengths of objects there is a need to use standard units. This exploration can then be used as a basis for discussing standard units of measure such as an inch. Rather than allowing students to explore measurement using a variety of concrete materials, measurement notions are introduced by the teacher through the use of a context. Then, the teacher is instructed to illustrate how she used a referent for one inch. Prior to this, no links are made to students' prior knowledge. Do the first grade students have the knowledge necessary to make sense of the use of a referent? The teacher is told to ask students how one might check the estimate and the

assumption is made that the students will suggest a ruler (e.g., “Children should suggest that you use a ruler.”). What happens if the students do not make such a suggestion? No guidance is provided to help teachers address unintended responses. Then the lesson moves to showing students that the string measures 12 inches using a ruler. If students do not have a good understanding of measurement prior to this, this illustration will likely have little meaning.

Regarding the use of representations, this lesson is problematic for several reasons. One, it does not provide the breadth of experiences with a variety of representations to allow students to make sense of what it means to measure. Two, students are moved to use a more abstract representation (i.e., the ruler) before they might be prepared to understand its purpose and use. Three, the lesson does not allow students to connect what they are learning to prior experiences, both inside and outside of school. A teacher who follows this lesson without making adaptations may fail to recognize the need to enhance students’ experiences by engaging in additional activity that could be used to support student learning. In sum, the use of representations was limited in this lesson. The lack of use of a variety of representations and the possible premature introduction of a representation that may be abstract to students could limit the development of students’ conceptual understanding.

### *Grade 3: Topic 6 – Fractions and Mixed Numbers – Frac Attack*

This lesson is part of the unit on fractions. If the recommended lesson sequence is followed, prior to the *Frac Attack* lessons, students would have had an opportunity to examine the meaning of fractions using various pictorial representations (i.e., set, length, and area models) as part of the Anchors lessons. In addition, they would have found equivalent fractions and compared fractions. The *Frac Attack* lesson is used to teach students how to add and subtract fractions. In this lesson, the use of representations is limited. First, students are not provided an opportunity to use the fraction knowledge they have gained in previous lessons to devise appropriate methods for adding or subtracting. Instead, students are told how to add and subtract fractions using only one representation, fractions strips. Second, students are not permitted to explore addition using the other representations that could be used to model other problems such as the set and area model. Next, links are made to symbolic representations (i.e., number sentences) without providing time for students to make sense of the mathematics through exploration with various representations or manipulatives. Overall, this lesson could have been approached from a more learner-centered perspective that encourages students to use prior knowledge and make connections to the mathematics under investigation. Rather than telling students what to do, they could be led to draw the same conclusions through exploration with a variety of representations.

In sum, the *Voyages* curriculum uses a wide range of representations to explore mathematics topics. The representations that are used can provide students insights to develop mathematics understanding. At times, the use of representations is limited. During particular lessons, we believe that students could benefit from the use of additional representations that allow them to compare and contrast the pros and cons of their use. The challenges related to the use of representations that were identified were primarily related to its implementation in the classroom rather than to its usage as part of the curriculum. Knowledgeable teachers can build on and enhance the lessons that are provided. In contrast, a teacher who is not knowledgeable could further limit students’ opportunity to learn by not exposing students to the representations provided in the curriculum, by not implementing the lessons in ways that encourage exploration, and by deciding to use lessons that provide direct instruction rather than a more open-ended learner-centered approach.

## *Technology*

According to the Standards, appropriate integration of technology is very important because “new technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them” (NCTM, 1989, p. 8). In addition, NCTM (2005) stated that [a] skillful teacher knows how to help students develop these abilities in a balanced program that focuses on mathematical understanding, proficiency, and thinking. The teacher should help students learn when to use a calculator and when not to, when to use pencil and paper, and when to do something in their heads. Students should become fluent in making decisions about which approach to use for different situations and proficient in using their chosen method to solve a wide range of problems. (p. 8)

After a careful review of the Anchors and Excursions materials, we found that the integration of technology was present, but not extensively. This presence was limited to some use of calculators. Furthermore, no rationale was provided for the use of technology. Grade 4 includes four technology related GLEs:

- MA.A.3.2.3.1: solves real-world problems involving addition, subtraction, multiplication, and division of whole numbers, and addition and subtraction of decimals and fractions using an appropriate method;
- MA.E.1.2.3.1: uses a calculator to determine the range and mean of a set of data;
- MA.E.1.2.3.2: uses computer applications to examine and evaluate data; and
- MA.E.1.2.3.3: uses computer applications to construct graphs.

However, only Anchors Lesson 10 (Topic 2: Deciding how to compute: Mental math, calculator, or paper and pencil) and Excursions lesson for group rate (Topic 4) were identified as related to technology. None of these activities explicitly included calculators as one of the needed materials. In addition, no instructions on how to use them within the procedures were provided. Similarly, Grade 4 included two Excursions related to technology, more specifically for the use of calculators. Calculators were listed as one of the needed materials for one of these Excursions lessons. One Excursions lesson for Grade 1, and four Anchors and four Excursions lessons for Grade 3 were identified as related to technology. None of these activities indicated the need for calculators as one of the materials, and did not explain how to use them within the procedures.

Although we recognize that calculator usage is not permitted on the FCAT, this should not limit the use of calculators when teaching mathematics. Calculators and other technology should be represented as another option to develop mathematical understandings (e.g., patterns) and as a tool to use when solving problems. Instructional software should also be included as part of curriculum procedures.

### **To What Extent does *Voyages* Address the New Florida Mathematics Standards to be Adopted in 2007?**

At the time of this writing, the new mathematics standards had not been formally adopted by Florida. The information provided below is based on the information made publicly available (see <http://www.flstandards.org>) and information garnered by two of the evaluators as part of the Standards development process.

## *About the Proposed Mathematics Standards*

The mathematics standards recommended for adoption in the State of Florida represent a significant departure from the Standards currently in use. The K-8 standards, in particular, follows closely the focal points recommended by the NCTM (2006) *Curriculum Focal Points* (CFP) (see Appendix B). (The full document is available at <http://www.nctm.org/standards/content.aspx?id=270>.) That is, important mathematics topics are identified for each grade level. Specifically, three curriculum focal points or “big ideas” and related connections for mathematics are provided for each grade level. CFP were developed to complement the Principles and Standards (NCTM, 2001), and as a reaction to criticisms that “the mathematics curriculum has been a mile wide and an inch deep.” One main assumption for the CFP is that when the curriculum focuses on a few key areas, students can be provided multiple and varied experiences that emphasize the core concepts and skills that facilitate the development of deep mathematical understanding, mathematics proficiency, mathematical fluency, and the ability to generalize and extend mathematical understanding. NCTM (2006) states the following:

To build students’ strength in the use of mathematical processes, instruction in these content areas should incorporate—

- the use of the mathematics to solve problems;
- an application of logical reasoning to justify procedures and solutions; and
- an involvement in the design and analysis of multiple representations to learn, make connections among, and communicate about the ideas within and outside of mathematics.

The purpose of identifying these grade-level curriculum focal points and connections is to enable students to learn the content in the context of a focused and cohesive curriculum that implements problem solving, reasoning, and critical thinking. (p. 10)

The proposed Florida mathematics for grades K-8 shares the same format as the CFP and is intended to support the same type of learning outcomes that is expected in the CFP. The organizational structure of the proposed mathematics standards is vastly different from the 1996 mathematics standards. In the 1996 Standards, many mathematics topics are revisited each year with the expectation that it will be studied at greater depth. In contrast, the proposed mathematics standards do not include this level of spiraling. Instead, the expectation is that at a particular grade level, the focus of instruction will be to fully develop students’ understanding of the key areas of emphasis. The curriculum at each grade-level includes fewer standards to learn, but the standards are expected to be learned with depth and breadth. That is, students are expected to fully understand the concepts. At the next grade level, students are expected to use the mathematics knowledge gained in the prior grade, but that content is not expected to be revisited as a major part of the curriculum. Given this arrangement of the curriculum, each teacher will have a greater impact on students’ mathematics development. Students who are not taught well will have little opportunity to revisit content that should have been taught thoroughly at a prior grade.

## ***Curricular Materials to Support the Implementation of Proposed Standards***

To date, there are no curricular materials commercially available to address the content and structure of the CFP or the proposed Florida mathematics standards. Because of the significant reorganization of the placement of topics to be learned, there would be a need to restructure available materials. However, even that would not be sufficient to address the depth of knowledge that is intended to be developed and supported. For example, if there were an attempt to simply identify a different sequence of lessons to follow pulling from lessons available in the K-5 *Voyages* materials, students would not develop the depth of understanding required by the new standards. In fact, there would likely be a great deal of repetition rather than extension. For example, a comparison of fractions lessons in Grade 4 and Grade 5 reveals that many of the same topics are addressed (e.g., parts of a whole, equivalent fractions, comparing fractions, adding and subtracting fractions, etc.). As a result, the curriculum would not necessarily provide additional materials to address the breadth of understanding that is intended. However, the Excursions lessons have the potential to allow teachers to develop students' understanding if they were extended, implemented in ways that are learner-centered, focused on authentic problem solving that allowed students to think mathematically and justify their thinking, and allowed students to explore and make mathematical connections through the use of multiple and varied representations.

### ***Reform-based Elementary School Curriculum***

Although not structured or organized in a manner to be readily used to support the proposed Florida mathematics standards, several elementary curricular materials exist that support the nature and spirit of the approaches recommended for teaching and learning mathematics by the mathematics community (NCTM 1989, 1991, 1995, 2001). Like *Voyages*, if used in their current form, there would be a need to restructure the materials to address the sequence of topics identified by the proposed standards. These research-based elementary curricular materials were developed with the support of funding from the National Science Foundation and were specifically designed to address the NCTM (1989) Standards. Florida's 1996 mathematics standards were based on these national standards.

- Investigations in Number, Data, & Space  
(<http://investigations.terc.edu/map/>)
- Everyday Mathematics  
(<http://everydaymath.uchicago.edu/educators/index.shtml>)
- Mathematics Trailblazers  
(<http://www.math.uic.edu/~imse/IMSE/MTB/mtb.html>)
- Mathematics Expressions  
(<http://eduplace.com/math/mthexp/>)

According to the Arc Center (<http://www.comap.com/elementary/projects/arc/>), a site that provides information about and supports the effective implementation of elementary-level standards-based curricula, all of these materials share the following assumptions about children's learning.

- Curricula should be conceptually oriented, supporting the child's development toward more abstract reasoning.

- Curricula should actively involve children in doing mathematics.
- Curricula should include a broad range of content -- not only arithmetic but also measurement, geometry, statistics, probability, and algebra.
- Curricula should make appropriate use of technology, including calculators and computers. (no page)

These materials are different from many available mainline curricula in that they:

- emphasize problem solving,
- emphasize depth of mathematics thinking,
- support the development of students' communication and reasoning,
- allow students to develop their own strategies,
- focus on the development of mathematics concepts, and
- develop mathematics understanding through the use of hands-on materials.

Each of these materials was developed over several years and involved a series of development, field testing, and refinement. The developers of each of these materials are engaged in ongoing research and evaluation of the effectiveness of their materials. Because of the nature of these curricula, the lessons provided within them are more likely to support the type of reasoning and knowledge development encouraged for Florida students.

### **Recommendations for SDHC Elementary Program**

Below we provide several comments and items to consider about the curriculum. In particular, we provide recommendations that should be considered to prepare for the implementation of the proposed Florida standards.

#### *Focus on More than Minimum Requirements*

While *Voyages'* aim is to cover all of the GLEs, in many instances the coverage is at the minimum level. For example, if a GLE indicates that students should study ordinal numbers up to 100th, the curriculum may only address ordinal numbers up to 10th. One might say that is it satisfactory for the student to study up to 10th and then just transfer knowledge to situations beyond 10th, but the challenge is not just to the 10th in a numerical sense but also understanding contexts that require the use of ordinal numbers beyond 10th.

To support students' mathematics development as intended in the proposed standards, curriculum and instruction must focus on the development of the depth of students' understandings. Students will be required to understand "why" a particular mathematical process is used in addition to understanding "how" to use that process. Students should be provided ample opportunity to engage in both problem solving and posing where they are asked to generate approaches to solve problem rather than mimic those presented by the teacher.

#### *Engage Students in Doing Mathematics with Opportunity for Exploration and Investigation*

In consideration of the phrase "to know mathematics is to do mathematics" one has to take note that the curriculum should provide students ample opportunity to engage in doing mathematics. The *Voyages* curriculum is poised to do this, but as currently written does not support this level

of engagement directly. For example, GLEs that promote students' building the groundwork for a concept by engaging in exploration, discussion, planning, presenting received no or limited attention or some lessons excluded these kinds of activities altogether.

Curriculum that is used to support the proposed curriculum needs to focus on and engage students in mathematics investigation and exploration. It is only through such engagement that students will develop the depth of knowledge expected of Florida's students.

#### *Include Approaches to Address the Needs of All Students*

The *Voyages* curriculum posits that it addresses the learning needs of all children. This is a high expectation but the curriculum fails to address this notion specifically. For example, the curriculum does not sufficiently accommodate the needs of English language learners (ELLs). There are no notable instances where the curriculum focuses on meanings of mathematical vocabulary that might assist ELLs in acquiring mathematical understanding. In addition, the contexts of lessons are essentially void of opportunity to make room for diverse perspectives on real-life.

The curriculum or supplemental materials to support the proposed curriculum need to identify and address the needs of students (e.g., ELLs, students with gaps in their knowledge) directly. This will be particularly important given the structure of the proposed curriculum. Students who are not provided opportunities to learn particular mathematics concepts or who transfer into the school district may miss some aspects of the curriculum that were designated to be learned in the prior grade. Because the curriculum does not spiral, it cannot be assumed that the students will pick it up in the next grade. Because of this, plans need to be in place to provide students timely support as needed.

#### *Encourage the Effective Use of Representation as Part of Mathematics Teaching and Learning*

The curriculum should provide more consistent use of different modes of representation (concrete level by using manipulative and real-life materials; representational level by using graphs, number lines, pictures, drawings and models; and abstract level by using words, definitions, explanations and symbols). This approach will meet all students' learning styles. In addition, this approach will provide various entry and access points to learning mathematics.

#### *Use Technology to Support Development of Mathematics Understanding*

It is highly recommended that technology, especially calculators, be integrated into the curriculum. This incorporation of technology should take into consideration the NCTM guidelines and research findings. There are various tools that can be used to enhance students' understanding of mathematics. For example, the Geometer's Sketchpad is one of the powerful software programs that provide opportunities for explorations and discovery of mathematics ideas. Virtual manipulatives and applets (i.e., computer simulations) are readily available on the Internet for instructional use and provide another strong source of technology application.

### *Do Not Limit the Scope of the Curriculum in an Attempt to Align it to the Standards and High Stakes Test*

The curriculum should take into account and align with state and national curriculum standards without losing track of the interconnection of mathematical ideas and skills. This interconnection should provide students with a deeper and more lasting understanding of mathematics, and a rich interplay among mathematical topics, in contexts that relate mathematics to other subjects and to their own interests and experience. The curriculum should not present mathematics as a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Instead, mathematics should be viewed as an integrated field of study (NCTM, 2001).

In some cases, if developed further and more connections are provided, many Excursions lessons that are part of the *Voyages* curriculum have the potential to provide more content interconnections. However, this approach needs to be enhanced, and with more consistency and clear purpose. Regarding the proposed curriculum, students will be expected to see connections among mathematics ideas.

### *Help Teachers Understand Frameworks that Underlie the Curriculum*

The *Voyages* curriculum posits that it addresses Gardner's theory of multiple intelligences. However, limited information is provided to support a thorough understanding of what is meant. Without professional development to support teachers and help them understand this theory, it is inadequate to simply list these concepts in the beginning of the curriculum and hypothesize that teachers will know how to activate them while they provide instruction.

Theories that support the development of the curriculum or that should be used as a basis of instruction should be thoroughly explained so that teachers are able to incorporate them as part of their instruction. Without such explanation or clarification either within the curriculum or as part of professional development, teachers are not provided with resources to support their implementation of the curriculum.

### *Provide Well-Structured and Ongoing Professional Development that Directly Supports Curricular Implementation*

In many school districts, teachers are offered a menu of professional development opportunities that they elect to attend. Though this structure provides opportunity for professional development, it does not necessarily address the type of learning that teachers need to enhance their instruction or encourage the effective implementation of the intended curriculum. Professional development that is offered should:

- help teachers understand the framework that was used in the development of the curriculum
- engage teachers in the exploration of mathematics so that they themselves develop conceptual understanding of the mathematics they are expected to teach.
- expose teachers to mathematics beyond the content that they are required to teach so that they recognize the role that the knowledge they are intended to develop plays in students' long term mathematics development.

- allow teachers to see direct connections to the intended curriculum and to the curriculum materials that are recommended for use.
- expose teachers to research findings so that they are able to implement instruction based on what is known about how children learn, misconceptions they have, or approaches that have been found to be effective to teach particular topics.
- engage teachers in the examination of students' thinking by viewing videos of students engaged in "doing" mathematics (e.g., Cognitively Guided Instruction) or through the examination and analysis of student work.
- individualize professional development so that it takes into account teachers' prior knowledge and experiences so that teacher knowledge continues to grow.

### **Final Remarks**

Although the curriculum materials selected for implementation play an important role in what students "know and are able to do," teachers' preparedness to use such materials provides a greater indication of whether students will have the opportunity to learn the curriculum as intended. To encourage the effective implementation of the curriculum, it is essential that teachers are provided professional development with follow-up and ongoing support. The professional development should help teachers' understanding of the theories that underlie the curriculum and develop their content and pedagogical content knowledge. Pedagogical content knowledge (as opposed to general pedagogical knowledge) focuses on pedagogy that is appropriate when teaching a particular subject matter, in this case mathematics. It includes the "most useful forms of representation of these ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1987, p. 9).

Simply making professional development available is not enough. Provided professional development should consider and differentiate teachers' backgrounds and needs. That is, a "one size fit all" approach will not likely be effective. Some elementary teachers will need to focus on mathematics content knowledge. Others will need greater attention to developing their pedagogical skills. Some teachers will need to develop both content and pedagogical content knowledge.

Provided professional development must be structured so that teachers are led to greater levels of understanding. Rather than providing a menu of offerings, teachers might be encouraged to participate in a well-structured sequence of professional development offerings that is designed to help them do something "specific" better with an expectation of use. Participation in subsequent sessions should be dependent on the successful completion of a prerequisite session or other experiences.

Teachers should be supported in their implementation of new curriculum materials or new instructional strategies. Teachers who participate in a professional development session may believe they understand presented strategies or approaches, but might be challenged to implement such strategies or approaches in their own classrooms. Without access to classroom level support, teachers may be discouraged and discontinue their use. However, classroom level support that involves a teacher's own students can be an effective means to help teachers change and improve instructional practices.

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