Overview:
This unit models real-world situations by using one- and two-variable linear equations. This unit will further expand upon previous knowledge of linear relationships by way of inverse functions, composite functions, piecewise-defined functions, operations on functions, and systems of linear equations and inequalities.

Standards:
Standards in this Unit:

MAFS.912.A-CED.1.2  Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MAFS.912.A-CED.1.3  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

MAFS.912.F-BF.2.3  Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

MAFS.912.F-BF.2.4  Find inverse functions.
  a.  Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2 x³ or f(x) = (x+1)/(x–1) for x ≠ 1.
  b.  Verify by composition that one function is the inverse of another.
  c.  Read values of an inverse function from a graph or a table, given that the function has an inverse.
  d.  Produce an invertible function from a non-invertible function by restricting the domain.

MAFS.912.F-IF.3.7a  Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  a.  Graph linear and quadratic functions, and show intercepts, maxima and minima.
  b.  Graph square root, cube root, and piece-wise functions, including step functions and absolute value functions.

MAFS.912.F-BF.1.1  Write a function that describes a relationship between two quantities.
  a.  Determine an explicit expression, a recursive process, or steps for calculation from a context.
  b.  Combine standard function types using arithmetic operations.
  c.  Compose functions.

MAFS.912.A-REI.3.6  Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (Tasks are limited to 3 by 3 systems in Algebra 2.)
Embedded Assessment 1:

Equations, Inequalities, and Systems – 
Gaming Systems (page 55)
Write an equation to represent a real-world scenario. 
(Lesson 1-2)
Graph an equation to represent a real-world scenario. 
(Lesson 2-1)
Evaluate an equation. (Lesson 1-2)
Write a system of inequalities to model given real-world constraints. (Lesson 1-3)
Graph a system of inequalities to model constraints. (Lesson 2-2)
Shade the solution region on a graph that is common to all of the inequalities. (Lesson 2-2)
Identify an ordered pair that satisfies constraints on a graph. (Lesson 2-2)
Explain what an ordered pair represents in the context of a situation. (Lesson 2-2)
Write an absolute value equation given real-world constraints. (Lesson 1-3)
Solve an absolute value equation. (Lesson 1-3)
Interpret the solutions to an absolute value equation. (Lesson 1-3)
Write a system of three equations to represent a real-world scenario. (Lesson 3-2)
Solve a system of three equations. (Lesson 3-2)
Interpret the solution to the system. (Lesson 3-2)

Embedded Assessment 2:

Piecewise-Defined, Composite, and Inverse Functions – Currency Conversion (page 99)
Define the units which represent the domain and range of a real-world scenario. (Lesson 4-1)
Convert quantities. (Lesson 5-1)
Write and explain a composition of functions (Lesson 5-2)
Identify the domain and range of composed functions (Lesson 5-2)
Use an inverse function. (Lesson 6-1)
Write a piecewise-defined function. (Lesson 4-1)
Write the domain of a piecewise-defined function using an inequality, interval notation and set notation. (Lesson 4-1)
Write the range of a piecewise-defined function using set notation. (Lesson 4-1)
Graph a piecewise-defined function (Lesson4-2)
Find the inverse of functions (Lesson 6-2)
Graph an absolute value function. (Lesson 4-3)
Describe the graph of a transformation of an absolute value graph. (Lesson 4-3)
Interpret an absolute value graph (Lesson 4-3)

Vocabulary (Academic/Math):
Academic vocabulary: interpret, compare, contrast, feasible, confirm, prove
Math vocabulary: absolute value equation, absolute value inequality, constraints, consistent, inconsistent, independent, dependent, ordered triple, Gaussian elimination, matrix, dimensions of a matrix, square matrix, multiplicative identity matrix, multiplicative inverse matrix, matrix equation, coefficient matrix, variable matrix, constant matrix, piecewise-defined function, step function, parent function, composition, composite function, inverse function
Learning Targets for lesson are found on page 3.

Main Ideas for success in Lesson 1-1:

⇒ Create an equation in one variable from a real-world context.
⇒ Solve an equation in one variable.
⇒ Vocabulary used in this lesson includes: algebraic expression, evaluate, interpret
⇒ Standards for Mathematical Practice to be demonstrated are: Reason Abstractly And Quantitatively, Make Sense Of Problems, Model With Mathematics

This example is a model to help solve Practice problems #14-15.

EXAMPLE: When full, a water storage tank holds 17,000 gallons of water. The tank currently holds 9,000 gallons of water and is being filled at a rate of 34 gallons per minute.

Part 1 - Write an equation that can be used to find $h$, the number of hours it will take to fill the tank from its current level. Explain the steps you used to write your equation.

Solution:

The storage tank holds 17,000 gallons of water and is starting off at a level of 9,000 gallons of water. We must figure out how many hours it will take to fill the reminder of the tank. Since the problem said that the tank is filled at a rate of 34 gallons per minute and we are told to use $h$ to represent the number of hours we must take into account that there are 60 minutes in 1 hour. We will multiply 34, the number of gallons filled per minute, by 60, the number of minute in an hour. $34(60h) + 9,000 = 17,000$.

60 multiplied by $h$ will represent the total number of minutes necessary to fill the remainder of the tank. This is multiplied by 34 because for each minute that passes, 34 gallons of water are added to the tank. $9,000$ is added to this value because it is an initial amount that is already sitting in the tank.

When we add both the product of $34(60h)$ and the 9,000 it will equal the total capacity (17,000) of the water storage tank.

Part 2 - Solve your equation from above, and interpret the solution.

Solution:

$34(60h) + 9,000 = 17,000$ (Original)

$2,040h + 9,000 = 17,000$ (Distributive property)

$2,040h + 9,000 - 9,000 = 17,000 - 9,000$ (Subtraction property of equality)

$2,040h = 8,000$ (Combine like terms)

$\frac{2,040h}{2,040} = \frac{8,000}{2,040}$ (Division property of equality)

$h \approx 3.9$

To interpret my answer I want to describe what my value of 3.9 represents in terms of my problem situation. It will take approximately 3.9 hours to fill the remainder of the water storage tank to capacity.
EXAMPLE The movie theater is open 7 days a week. The theater has 10 ticket booths, and each booth has a ticket seller from 11:00am to 10:00pm. On average, ticket sellers work 25 hours per week. This situation will be used for all examples to follow.

Write an equation that can be used to find \( t \), the minimum number of ticket sellers the theater needs. Explain the steps you used to write your equation.

Solution:

\[ 25t = 77(10) \]

The minimum number of ticket sellers, \( t \), times 25 hours per week equals the total number of hours worked by the ticket sellers per week. This needs to equal the total number of hours the theater is open. The problem stated that each of the 10 booths is manned at all times. If the theater is open from 11:00am to 10:00pm, it is open for 11 hours per day and since it is open 7 days a week that means the theater is open for a total of 77 hours per week (11 times 7).

EXAMPLE Solve the equation from above and interpret the solution. Refer to the above example.

Solution:

\[
\begin{align*}
25t &= 770 \\
&= 770 \\
\frac{25t}{25} &= \frac{770}{25} \\
t &= 30.8
\end{align*}
\]

Since \( t \) represents the minimum number of ticket sellers, \( t \) must be a whole number, it represents a number of people. It is not okay to round down to 30 because then we would be short a ticket seller at some point throughout the week. Therefore, we must round up to 31 to ensure that we will have enough. 31 is the fewest number of ticket sellers the theater needs.

EXAMPLE The theater plans to hire 15% more than the minimum number of ticket sellers needed in order to account for sickness, vacation, and lunch breaks. How many ticket sellers should the park hire? Explain. Refer to the above example.

Solution:

We found earlier that the minimum number of ticket sellers needed was 31. We must now find what 15% of 31 is. I must convert 15% to its equivalent decimal by dividing 15 by 100 to get 0.15 and then multiply it by our 31 people.

\[ 31 \cdot 0.15 = 4.65 \]

Again, we want to round up to the next whole number since we are representing people. Therefore the theater will want to add an additional 5 people. The theater will then be hiring a total of 36 ticket sellers (31 + 5).
Additional Practice:

Use this information for Items 1–3. Aaron has $65 to rent a bike in the city. It costs $15 per hour to rent a bike. The additional fee for a helmet is $3 for the entire ride.

1. Write an equation that can be used to find \( h \), the number of hours Aaron can rent the bike.

2. Solve your equation from Item 1, and interpret the solution.

3. Aaron would like to rent the bike for 5 hours. How much more money will Aaron need? Explain.

4. Aaron and Zelly want to rent a tandem bike so that they can ride together. The rental for a tandem bike is $18.50 per hour plus $3 for each helmet. Aaron and Zelly have $80 to spend on the bike rental. Which equation can be used to find \( h \), the number of hours they can rent a tandem bike?
   
   A. \( 80 = 18.50h + 3 \)  
   B. \( 80 = 18.50h + 6 \)  
   C. \( 80 = 3h + 18.50 \)  
   D. \( 80 = 6h + 18.50 \)

5. Make sense of problems. Eliza bought a day pass to rent a bike in the city. The day pass costs $40 from 9 a.m. to 7 p.m. There is an additional fee of $4 per quarter hour if she returns the bike after 7 p.m. Eliza has $50 and plans to return the bike at 8:15 p.m.
   
   a. Does Eliza have enough money? Explain using an equation.

   b. When should she return the bike?

Answers to additional practice:

1. \( 65 = 15h + 3 \)
2. 4 hours; the cost of renting a bike for 4 hours is $63.
3. $13; it costs $78 to rent the bike for 5 hours since 15(5) + 3 = 78. This is $13 more than Aaron has, 78 - 65 = 13.
4. B
5. a. No, there are 5 quarter-hour segments from 7:00 p.m. to 8:15 p.m. Using the equation \( c = 40 + 4(5) \), c is $60.
   
   b. Eliza should return the bike by 7:30 p.m. since 40 + 4(2) = 48. She has enough money to keep the bike until 7:30 p.m.
Learning Targets for lesson are found on page 7.

Main Ideas for success in 1-2:

⇒ Create equations in two variables to represent relationships between quantities.
⇒ Graph two-variable equations.
⇒ Vocabulary used in this lesson includes: independent variable, dependent variable, linear equation, y-intercept, slope
⇒ Standards for Mathematical Practice to be demonstrated are: Reason Abstractly, Construct Viable Arguments, Reason Quantitatively.

Practice Support for Lesson 1-2

⇒ This example is a model to help solve Practice problem 18.

**EXAMPLE:** The water storage tank is periodically emptied and cleaned to prevent any algae growth. The label on the chemical states to add 4 fluid ounces per 1,000 gallons of water.

Make a table that shows how much of the chemical to add for other water tanks holding 5,000; 10,000; 15,000; and 20,000 gallons of water.

Solution:

<table>
<thead>
<tr>
<th>Gallons of water</th>
<th>Process</th>
<th>Fluid ounces of chemical</th>
</tr>
</thead>
</table>
| 1,000            | \[
\frac{4 \text{ fluid ounces}}{1,000 \text{ gallons}} = \frac{x \text{ fluid ounces}}{1,000 \text{ gallons}}
\]
\[
(4) \cdot (1,000) = (1,000) \cdot (x)
\]
\[
\frac{4,000}{1,000} = \frac{1,000x}{1,000}
\]
\[
4 = x
\] | 4 fluid ounces |
| 5,000            | \[
\frac{4 \text{ fluid ounces}}{1,000 \text{ gallons}} = \frac{x \text{ fluid ounces}}{5,000 \text{ gallons}}
\]
| 40 fluid ounces |
| 10,000           | \[
\frac{4 \text{ fluid ounces}}{1,000 \text{ gallons}} = \frac{x \text{ fluid ounces}}{10,000 \text{ gallons}}
\]
| 60 fluid ounces |
| 15,000           | \[
\frac{4 \text{ fluid ounces}}{1,000 \text{ gallons}} = \frac{x \text{ fluid ounces}}{15,000 \text{ gallons}}
\]
| 80 fluid ounces |
| 20,000           | \[
\frac{4 \text{ fluid ounces}}{1,000 \text{ gallons}} = \frac{x \text{ fluid ounces}}{20,000 \text{ gallons}}
\]
⇒ This example is a model to help solve Practice problem 19.

**EXAMPLE:** Write a linear equation in two variables that models the situation above. Tell what each variable in the equation represents. Refer to the above example.

Solution:

\[y = 0.004x\]

Since the problem states that we need to pour 4 fluid ounces of the chemical in for every 1,000 gallons that is equivalent to 0.004 gallons of the chemical for every 1 gallon of water. This is from dividing 4 by 1,000. X represents the number of gallons the water tank holds, input. Y represents the total number of fluid ounces of the chemical to be added, output.
This example is a model to help solve Practice problem 20.

**EXAMPLE:** Graph the equation. Be sure to include titles and use an appropriate scale on each axis. Refer to the example above.

**Solution:**

Using the table from earlier, we designated $x$ as being the amount of water in a storage tank and $y$ as being the amount of chemical added in fluid ounces to the water tank. We can take these values from our table and plot them as ordered pairs $(x, y)$: $(1,000, 4), (5,000, 20), (10,000, 40), (15,000, 60), (20,000, 80)$. This will represent the line of our equation.

This example is a model to help solve Practice problem 21.

**EXAMPLE:** What is the slope and $y$-intercept of the graph? What do they represent in the situation? Refer to the above example.

**Solution:** Our equation from above is in slope-intercept form therefore we can easily identify the slope and $y$-intercept. Slope intercept form is $y = mx + b$ where $m$ is the slope and $b$ is the $y$-intercept. Since our equation is $y = 0.004x$, we know that the slope is 0.004 and the $y$-intercept is 0, because there is no constant term at the end of our equation. A slope of 0.004 means that for each additional gallon of water the tank holds we need to add in 0.004 fluid ounces of the chemical. A $y$-intercept of 0 means that when the amount of water in the tank is 0 gallons that we need to add in 0 fluid ounces of the chemical.

This example is a model to help solve Practice problem 22.

**EXAMPLE:** An employee adds 62 fluid ounces of the chemical to a water storage tank that holds 16,000 gallons of water. Did the employee add the correct amount? Explain.

**Solution:** No, substituting 16,000 for $x$ in the equation and solving for $y$ shows that the employee should have added 64 fluid ounces of the chemical.

- $y = 0.004x$ equation
- $y = 0.004(16000)$ evaluate when $x = 16,000$
- $y = 64$ amount of fluid ounces of the chemical to be added
Additional Practice:

1. Make a table that shows the cost of the rental for 1, 2, 3, 4, and 5 days.

2. Write a linear equation in two variables that models the situation. Tell what each variable in the equation represents.

3. Graph the equation. Be sure to include titles, and use an appropriate scale on each axis.

4. Which of the following statements describes what the slope and y-intercept represent in the situation?
   A. The slope represents the cost of the rental. The y-intercept represents the cost of the helmet.
   B. The slope represents the cost of the helmet per day. The y-intercept represents the cost of the helmet at day 0.
   C. The slope represents the cost of the bike per day. The y-intercept represents the cost of the helmet for the entire time of the rental.
   D. The slope represents the rise of a hill in New Orleans. The y-intercept represents rise of the slope.

5. Construct viable arguments. Barbara has $250 to rent a bike for 15 days. Does Barbara have enough money? Explain.

Answers to Additional Practice:

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$27</td>
</tr>
<tr>
<td>2</td>
<td>$47</td>
</tr>
<tr>
<td>3</td>
<td>$67</td>
</tr>
<tr>
<td>4</td>
<td>$87</td>
</tr>
<tr>
<td>5</td>
<td>$107</td>
</tr>
</tbody>
</table>

Sample answer: \( c = 20d + 7 \); \( c \) represents the cost of the bike rental, and \( d \) represents the number of days Susan rents the bike.

No; sample answer: Substituting 15 for \( d \) in the equation and solving for \( c \) shows that Barbara does not have enough money to rent the bike for 15 days.
**Support for Lesson 1-3**

**Learning Targets** for lesson are found on page 11.

**Main Ideas for success in 1-3:**

⇒ Write, solve, and graph absolute value equations.
⇒ Solve and graph absolute value inequalities.
⇒ Vocabulary used in this lesson includes: absolute value equation, absolute value inequality, compare, contrast
⇒ Standards for Mathematical Practice to be demonstrated are: Reason Abstractly, Reason Quantitatively, Make Sense Of Problems, Critique The Reasoning Of Others, Model With Mathematics

**Practice Support for Lesson 1-3**

⇒ This example is a model to help solve Practice problems 6 and 7.

**EXAMPLE:** Solve the absolute value equation \( |4x - 8| = 30 \)

\[
|4x - 8| = 30 \quad \text{original (the absolute value expression is already isolated)}
\]

\[
4x - 8 = 30 \quad \text{or} \quad 4x - 8 = -30 \quad \text{write as two equations using the definition of absolute value}
\]

\[
4x = 38 \quad \text{or} \quad 4x = -22 \quad \text{add 8 to both sides}
\]

\[
x = \frac{19}{2} \quad \text{or} \quad x = -\frac{11}{2} \quad \text{divide both sides by 4}
\]

There are two solutions: \( x = \frac{19}{2} \) and \( x = -\frac{11}{2} \).

⇒ This example is a model to help solve Practice problems 8 and 9.

**EXAMPLE:** Solve the absolute value equation \( |2x - 4| + 10 = 5 \)

\[
|2x - 4| + 10 = 5 \quad \text{original}
\]

\[
|2x - 4| = -5 \quad \text{isolate the absolute value expression by subtracting 10 on both sides}
\]

No solution

Since the absolute value of any expression will equal a positive value, there is no solution. There is no value to plug in for \( x \) that will provide us with an output of -5.

⇒ This example is a model to help solve Practice problems 10.

**EXAMPLE** The Little River Dam has a flow rate that can vary up to 74 gallons per minute from the target flow rate of 1,200 gallons per minute. Write and solve an absolute value equation to find the extreme values of the flow rate for the Little River Dam.

\[
|f - 1,200| = 74 \quad \text{original}
\]

\[
f - 1,200 = 74 \quad \text{or} \quad f - 1,200 = -74 \quad \text{use the definition of absolute value to make two equations.}
\]

\[
f = 1,274 \quad \text{or} \quad f = 1,126 \quad \text{add 1,200 to both sides.}
\]

The two extreme values that represent the flow rate for the Little River Dam are 1,126 and 1,274 gallons per minute.
Practice Support for Lesson Continued

This example is a model to help solve Practice problems 11 and 12.

**EXAMPLE:** Solve the absolute value inequality. Graph the solution on a number line. \( |3x - 6| > 6 \)

- \( 3x - 6 > 6 \) or \( 3x - 6 < -6 \) write two inequalities (notice the change in inequalities and signs)
- \( 3x > 12 \) or \( 3x < 0 \) add 6 to both sides
- \( x > 4 \) or \( x < 0 \) divide both sides by 3

This example is a model to help solve Practice problem 13.

**EXAMPLE:** Solve the absolute value inequality. Graph the solution on a number line. \( |2x - 10| - 4 > 8 \)

- \( |2x - 10| > 12 \) add 4 to both sides to isolate the absolute value expression
- \( 2x - 10 > 12 \) or \( 2x - 10 < -12 \) write two inequalities (note sign changes)
- \( 2x > 22 \) or \( 2x < -2 \) add 10 to both sides
- \( x > 11 \) or \( x < -1 \) divide both sides by 3

This example is a model to help solve Practice problem 14.

**EXAMPLE:** Solve the absolute value inequality. Graph the solution on a number line. \( |2x - 4| - 10 \leq 8 \)

- \( |2x - 4| \leq 18 \) add 10 to both sides to isolate the absolute value expression
- \( -18 \leq 2x - 4 \leq 18 \) write the compound inequality
- \( -14 \leq 2x \leq 22 \) add 4 to all parts
- \( -7 \leq x \leq 11 \) divide by 2 on all parts
Additional Practice:

1. Solve each absolute value equation. Graph the solutions on the number line.
   a. \(5|x - 6| = 20\)
      \[\begin{array}{c}
      -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \end{array}\]
   b. \(|3x + 15| = 9\)
      \[\begin{array}{c}
      -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \end{array}\]

2. If the speed of a car on the highway detected by a radar gun is within 7 mph of the speed limit, a police officer will not issue a ticket to a car for driving too fast or too slow. What is the acceptable range of speeds in a 55-mph speed zone?

3. Solve each absolute value inequality. Graph the solutions on the number line.
   a. \(|x - 5| < 4\)
      \[\begin{array}{c}
      -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \end{array}\]
   b. \(|3x + 5| \geq 8\)
      \[\begin{array}{c}
      -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \end{array}\]
   c. \(5|2x + 9| > 15\)
      \[\begin{array}{c}
      -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \end{array}\]

4. Model with mathematics. The weight of a pen manufactured at the PEN factory is 8 g. The actual weight can vary by as much as 2 g. Write and solve an inequality to represent all acceptable weights for a pen at this factory. Graph the inequality.

Answers to additional practice:

1a.
   \(c = 2, x = 10\)
   \[\begin{array}{c}
   -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
   \end{array}\]

1b.
   \(d = -8, x = -2\)
   \[\begin{array}{c}
   -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
   \end{array}\]

2.
   \(48 \leq s \leq 62\)

3.
   a. \(1 < x < 9\)
      \[\begin{array}{c}
      -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
   \end{array}\]
   b. \(x \leq -4\text{ or } x \geq 1\)
      \[\begin{array}{c}
      -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
   \end{array}\]
   c. \(< -6 \text{ or } x > -3\)
      \[\begin{array}{c}
      -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
   \end{array}\]

4.
   \(2| \leq 8; 6 \leq w \leq 10;\)
   \[\begin{array}{c}
   -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
   \end{array}\]
Support for Lesson 2-1

Learning Targets for lesson are found on page 17.

Main Ideas for success in 2-1

⇒ Write equations in two variables to represent relationships between quantities.
⇒ Graph equations on coordinate axes with labels and scales.
⇒ Vocabulary used in this lesson includes function, rate of change, and slope intercept form.
⇒ Standards for Mathematical Practice to be demonstrated are reason quantitatively, model with mathematics, and make sense of problems

Practice Support for Lesson 2-1

⇒ This example is a model to help solve Practice problems 15-20.

EXAMPLE:

⇒ This example is a model to help solve Practice problem 15.

1. Write the equation of the line with y-intercept -2 and a slope of \( \frac{4}{3} \). Graph the equation.

Since you are given the slope and the y-intercept you can plug the values directly into slope intercept form: 

\[ y = mx + b. \]

Remember \( m \) represents the slope and \( b \) represents the y-intercept, so \( m = \frac{4}{3} \) and \( b = -2 \). Plug the \( m \) and \( b \) values into the equation. 

\[ y = \frac{4}{3}x - 2. \]

2. When graphing BEGIN with the \( b \) value. Since -2 represents the y-intercept, I will move down two units on the y axis and plot a point. Slope (\( m \)) is what we MOVE from the y-intercept. Slope represents the rise over the run, so the numerator is what you move up or down (rise), and the denominator is what you move left or right (run). Since our slope is positive, we will move up four and right 3 from our y-intercept of -2.

A quick check to make sure your answer is reasonable: positive slopes should rise from left to right, and negative slopes should fall from left to right.
This example is a model to help solve Practice problem 16.

**EXAMPLE**

Write the equation of the line that through the point (-1,-4) with a slope of 2. Graph the equation.

Use the coordinate and the slope to find the y-intercept. In the coordinate (-1,-4), -1 is an x value and -4 is a y value. Remember from the previous example that the slope 2 will be $m$.

Start with slope-intercept form $y = mx + b$

Plug in all given values: $y=-4$, $m=2$, and $x=-1$. The only unknown value in the equation is $b$ or the y-intercept.

$-4 = 2(-1) + b$ 

Isolate $b$

$-4 = -2 + b$

$-2 = b$

Substitute $b$ back into slope-intercept form with the given slope.

$y = 2x - 2$

Graph $y = 2x - 2$

BEGIN at y-intercept: -2 ($b$)

MOVE $\frac{\text{rise}}{\text{run}} = \frac{2}{1} (m)$ check to make sure your line and your slope match. (Positive slope should rise from left to right)

This example is a model to help solve Practice problem 17.

**EXAMPLE:**

Graph the function $f(x) = 2 - \frac{1}{3}(x - 9)$. $F(x)$ is the function notation for $y$.

So replace $f(x)$ with $y$. $y = 2 - \frac{1}{3}(x - 9)$

Get into slope-intercept form

$y = 2 - \frac{1}{3}x + 3$ Distribute the $-\frac{1}{3}$
$y = - \frac{1}{3}x + 5$ Combine like terms

Slope ($m$) = $- \frac{1}{3}$ and y-intercept ($b$) = 3

BEGIN at the y-intercept 3 and MOVE the slope $- \frac{1}{3}$. Make sure when you are moving the slope from the y-intercept that you only move ONE direction negative, not both numbers in the negative direction. For example you could move up one and left three or down one and right three but NOT down one and left three. Two negatives make a positive.

Graph

⇒ This example is a model to help solve Practice problems 18

**EXAMPLE:**

Rachel already has 8,524 frequent flyer miles, and she will earn 1,776 more miles from her round-trip flight to Seattle. In addition, she earns 1 frequent flyer mile for each dollar she charges on her credit card.

Write the equation of a function $f(d)$ that represents the total number of frequent flyer miles Rachel will have after her trip if she charges $d$ dollars on her credit card.

What does Rachel already have? 8,524 frequent flyer miles and she is about to get 1,776 more for her Seattle trip. So those will be added together $8,524 + 1,776 = 10,300$ frequent flyer miles

How can she get additional miles? 1 mile per dollar charged on her credit card. The amount of miles is dependent on the number of dollars charged, so $1d$ or just $d$.

So the function modeling her frequent flyer miles after her trip would be: $f(d) = 10,300 + d$

⇒ This example is a model to help solve Practice problems 19

**EXAMPLE:**

Graph $f(d) = 10,300 + d$ which is the same as $f(d) = d + 10,300$. Identify the slope and the y-intercept so you can graph the function. The slope is 1, the coefficient of $d$, and the y-intercept is 10,300.
The numbers on the x-axis represent the dollars Rachel charged on her credit card. The dollars could be counted in several different increments, but it would be good to count by

⇒ This example is a model to help solve Practice problems

**EXAMPLE**
EXAMPLE:

Reason quantitatively. How many dollars will Rachel need to charge on her credit card to have a total of 13,000 frequent flyer miles? Explain how you determined your answer.

Use the equation created in number 18: \( f(d) = 10,300 + d \) to solve this question.

\( d \) is the number of dollars charged on Rachel’s credit card and \( f(d) \) is the number of frequent flier miles given the dollars spent. So for this problem 13,000 will be substituted for \( f(d) \) and \( d \) will be our unknown value.

\[
13,000 = 10,300 + d \\
2,700 = d
\]

This means that when Rachel charges $2700 on her credit card, she will have 13,000 frequent flier miles.
**Additional Practice:**

16. Write the equation of a line with a y-intercept of 2 and a slope of $\frac{3}{4}$. Graph the equation.

17. Which is the equation of the line that passes through the point $(2, -2)$ and has a slope of $3$?
   - A. $y = 3x - 2$
   - B. $y = 3x - 8$
   - C. $y = 2x + 3$
   - D. $y = -2x + 2$

Use the following information for Items 18-20. Julian works as a sales representative for a home improvement store. He earns $1200 plus $9.5\%$ commission on sales each month.

18. Write the equation of the function $f(x)$ that represents Julian’s monthly earnings, where $x$ represents the amount of monthly sales.

19. Graph the function using appropriate scales on the axes.

20. **Reason quantitatively.** What would Julian’s sales have to be for him to earn $3200$ in one month? Explain how you determined your answer.

**Additional Practice Answers:**

16. $y = \frac{3}{4}x + 2$.

17. **B**

18. $f(x) = 0.095x + 1200$

19. 

20. $\$8421.05$; solve the equation $2000 = 0.095s + 1200$ for $s$. 
Learning Targets for lesson are found on page 21.

Main Ideas for success in 2-2:

⇒ Represent constraints by equations or inequalities.
⇒ Use a graph to determine solutions of a system of inequalities.
⇒ Vocabulary used in this lesson includes inequalities, and constraints
⇒ Standards for Mathematical Practice to be demonstrated are construct viable arguments, attend to precision, use appropriate tools strategically, and model with mathematics.

Practice Support for Lesson 2-2

⇒ This example is a model to help solve Practice problem 19.

EXAMPLE:

Graph these inequalities on the same grid, and shade the solution region that is common to all of the inequalities:

\[ y \geq 1, \quad y \leq 1 + \frac{1}{3}x, \quad x \leq 4. \]

The solutions to these inequalities lie where the three shadings overlap.

1. Graph each inequality on the same graph and shade.
   A. \[ y \geq 1 \]
      \[ y = 1 \] graph shade the side of the line where the y values are greater than one. The line is solid because the coordinates on the line are also solutions to the inequality.

   B. \[ y \leq 1 + \frac{1}{3}x \]
      \[ y = \frac{1}{3}x + 1 \] Use the slope and the y-intercept to graph (as described in lesson 2-1)
      Shade where the y values are less than the graphed line.
C. $x \leq 4$

$x = 4$ Graph the line and shade where the $x$ values are less than 4

The final graph should look like this; the solutions lie where the shading overlaps in the highlighted triangle.

⇒ This example is a model to help solve Practice problem 20.

EXAMPLE

Using the final graph from question 19, choose two points from the highlighted region where all three shadings overlap. The coordinates in this region will make all three inequalities true. Remember since the lines are solid, that coordinates on the lines can be used. So (3,1) and (4,2) can be solutions.

⇒ This example is a model to help solve Practice problems 21.

EXAMPLE

A snack company plans to package a mixture of almonds and peanuts. The table shows information about these types of nuts. The company wants the nuts in each package to have at least 30 grams of protein and to cost no more than $2. Use this information for Items 21–23.

Model with mathematics. Write inequalities that model the constraints in this situation. Let $x$ represent the number of ounces of almonds in each package and $y$ represent the number of ounces of peanuts. Start by writing an inequality to represent the constraints on each of the following: protein, cost, $x$, and $y$. Remember almonds are $x$ and peanuts are $y$. Protein: $4x + 6y \geq 30$, Cost: $15x + .10y \leq 2$, Almonds $x \geq 0$ (can’t have negative almonds or peanuts) Peanuts $y \geq 0$
**Practice Support for Lesson Continued:**

⇒ This example is a model to help solve Practice problem 22.

**EXAMPLE**

Graph the constraints. Shade the solution region that is common to all of the inequalities.

Apply the same process as number 19. Graph and shade each inequality on the same graph and see where the shading overlaps. Replace inequality with an equal sign and get into slope-intercept form so you can graph line.

a. \(4x + 6y \geq 30\)  
   \(4x + 6y = 30\) Solve for \(y\)  
   \(6y = -4x + 30\)  
   \(y = -\frac{2}{3}x + 5\)

b. \(.15x + .10y \leq 2\)  
   \(.15x + .10y = 2\)

c. \(y = 0\)

d. \(x = 0\)

⇒ This example is a model to help solve Practice problem 23

**EXAMPLE:**

a. Identify two ordered pairs that satisfy the constraints. Pick any two points from the region where the shading overlaps. (2,6) and (4,4)

b. **Reason quantitatively.** Which ordered pair represents the more expensive mixture? Which ordered pair represents the mixture with more protein? Explain your answer.

Plug both points into both the cost and protein equation from number 21 to determine higher amount.

**Protein:**

\[
\begin{align*}
(2,6) & : & 4x + 6y &= 4(2) + 6(6) = 8 + 36 = 44 \text{g} \\
(4,4) & : & 4x + 6y &= 4(4) + 6(4) = 16 + 24 = 40 \text{g}
\end{align*}
\]

**Cost:**

\[
\begin{align*}
(2,6) & : & .15x + .10y &= .15(2) + .10(6) = .30 + .60 = .90 \\
(4,4) & : & .15x + .10y &= .15(4) + .10(4) = .60 + .40 = 1.00
\end{align*}
\]
<table>
<thead>
<tr>
<th>.15(2) + .10(6)</th>
<th>.15(4) + .10(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.30 + .60</td>
<td>.60 + .40</td>
</tr>
<tr>
<td>.90</td>
<td>1</td>
</tr>
</tbody>
</table>

After plugging in both points look to see which point produced the larger number. For protein, the coordinate (2,6) has 44 grams compared to (4,4) which only has 40 grams. So (2,6) represents the mixture with more protein. Looking at cost, (4,4) is one dollar while (2,6) is only 90 cents. Therefore (4,4) is the more expensive mixture.
Additional Practice:

21. a. Graph the inequality \(-2x - y < -5\).

25. **Model with mathematics.** Write a system of inequalities for the graph.

b. Did you use a solid or dashed line for the boundary line? Explain your choice.

c. Did you shade above or below the boundary line? Explain your choice.

22. Graph the following inequalities on the same grid and shade the solution region that is common to all of the inequalities.

   a. \(x < 2\)
   b. \(y \leq 2x\)
   c. \(y \geq -3x + 2\)

23. Which ordered pair lies in the solution region that is common to all of the inequalities in Item 22?

   A. (2, 5)  B. (0, 0)
   C. (2, 0)  D. (1, -2)

24. a. Identify two ordered pairs that do not satisfy the constraints in Item 22.

   a. Sample answer: \((-2, 1)\) and \((3, 6)\).
   b. Sample answer: \((-2, 1)\) does not satisfy the constraint \(y \geq -3x + 2\) and \((3, 6)\) does not satisfy the constraint \(x < 2\).

   25. \(x \geq 1, y < 2, y \geq \frac{1}{2}x - 2\)
**Support for Lesson 3-1**

**Learning Targets** for lesson are found on page 29.

**Main Ideas for success in 3-1**

⇒ Use graphing, substitution, elimination to solve systems of linear equations in two variables.
⇒ Formulate systems of linear equations in two variables to model real-world situations.
⇒ Vocabulary used in this lesson includes solutions of a system of linear equations, consistent, inconsistent, independent, dependent, substitution method, and elimination method.
⇒ Standards for Mathematical Practice to be demonstrated are reason abstractly and quantitatively, make sense of problems and persevere in solving them, model with mathematics, construct viable arguments, and make use of structure.

**Practice Support for Lesson 3-1**

⇒ This example is a model to help solve Practice problem 15.

**EXAMPLE:**

Solve the system by graphing. \[
\begin{align*}
y &= -2x - 1 \\
5x + 6 &= y
\end{align*}
\]

Get both equations into slope-intercept form in order to graph them. \[
\begin{align*}
y &= -2x - 1 \\
y &= 5x + 6
\end{align*}
\]

Use the slope and the y-intercept to graph each line on the same coordinate plane. Remember to begin by plotting the y-intercept, and move the slope \(\frac{\text{rise}}{\text{run}}\) from the y-intercept for each line.

The solution to any system is the point that is on both lines. The two lines intersect at (-1,1) making it the solutions to this system of equations.

⇒ This example is a model to help solve Practice problem 16.

**EXAMPLE:**

Solve the system using substitution. \[
\begin{align*}
5y + 5 &= x \\
7y - x &= -23
\end{align*}
\]

This method is good for solving a system when there is a variable with a coefficient of one. The top equation is already solved for \(x\), which has a coefficient of one. Substitute the \(5y + 5\) into the bottom equation where the \(x\) is.

\[
7y - (5y + 5) = -23 \quad \text{distribute negative}
\]

\[
7y - 5y - 5 = -23 \quad \text{combine like terms}
\]

\[
2y - 5 = -23 \quad \text{solve for y}
\]
Remember the solution to any system is a coordinate that satisfies both equations or is on both lines. So the answer should always be an (x,y) coordinate. Now that the y value is known, it can be used to solve for x. Plug the y value into either equation in order to find the value for x.

\[5(-9) + 5 = x\]
\[-45 + 5 = x\]
\[-40 = x\]

Write the answer as an ordered pair: (-40,-9)

EXAMPLE:

Solve the system by elimination \( \begin{cases} 3x - 2y = -21 \\ 2x + 5y = 5 \end{cases} \)

In elimination method, the goal is to eliminate one of the variables. This means that the coefficient must become zero; this occurs when the coefficients of one variable become the same number with the opposite sign. It does not matter which variable is chosen to eliminate. In this case both equations will have to be multiplied by a value in order to get a coefficient of zero. Let’s eliminate x in this example.

Multiply both equations in order to eliminate x:

\[ (3x - 2y = -21) \times 2 \quad \rightarrow \quad 6x - 4y = -42 \]
\[ (2x + 5y = 5) \times -3 \quad \rightarrow \quad -6x - 15y = -15 \]

Add new equations together:

\[ \begin{align*}
6x - 4y &= -42 \\
-6x - 15y &= -15
\end{align*} \]

Solve for y:

\[-19y = -57\]
\[y = 3\]

Plug y into one of the original equations:

\[3x - 2(3) = -21\]
\[3x - 6 = -21\]
\[3x = -15\]
\[x = -5\]

Write the answer as coordinate:

\((-5, 3)\)
Practice Support for Lesson Continued: 3-1

⇒ This example is a model to help solve Practice problem 18.

EXAMPLE

Make sense of problems and persevere in solving them. At one company, a level I engineer receives a salary of $30,000, and a level II engineer receives a salary of $42,000. The company has 9 level I engineers. Next year, it can afford to pay $282,000 for their salaries. Write and solve a system of equations to find how many of the engineers the company can afford to promote to level II.

Define variables and create two equations

Let \( x \) = level I engineers    Let \( y \) = level II engineers

Equation representing engineers: \( x + y = 9 \)

Equation representing salaries: \( 30,000x + 42,000y = 282,000 \)

Solve the system of equations using the method of your choice

(substitution method)

\[
\begin{align*}
  x &= -y + 9 \\
  30,000(-y + 9) + 42,000y &= 282,000 \\
  -30,000y + 270,000 + 42,000y &= 282,000 \\
  12,000y + 270,000 &= 282,000 \\
  12,000y &= 12,000 \\
  y &= 1
\end{align*}
\]

Remember we defined \( y \) as the number of level II engineers, so the company can promote 1 level one engineer.

⇒ This example is a model to help solve Practice problem 19.

EXAMPLE:

Answers may vary as there is no one correct way to solve a system. Substitution was the chosen method due to the fact that the coefficients were already one in the engineer’s equation.
26. Solve each system by graphing.
   a. \[
   \begin{align*}
   y &= -x \\
   y &= 2x + 3
   \end{align*}
   \]

   b. \[
   \begin{align*}
   y &= \frac{2}{3}x - 5 \\
   y &= \frac{2}{3}x
   \end{align*}
   \]

27. Solve each system by substitution.
   a. \[
   \begin{align*}
   3x - y &= 8 \\
   y &= 4 - x
   \end{align*}
   \]
   b. \[
   \begin{align*}
   3x - 5y &= 11 \\
   x - 3y &= 1
   \end{align*}
   \]
   c. \[
   \begin{align*}
   3x - y &= 3 \\
   \frac{3}{5}x - 5y &= 15
   \end{align*}
   \]
   d. \[
   \begin{align*}
   5x - y &= 5 \\
   4x - 5y &= -17
   \end{align*}
   \]
28. Solve each system by elimination.
   
   a. \[ \begin{align*}
   5x + 2y &= 6 \\
   9x + 2y &= 22
   \end{align*} \]
   
   b. \[ \begin{align*}
   4x - 2y &= 12 \\
   4x + 2y &= 24
   \end{align*} \]
   
   c. \[ \begin{align*}
   6x - 5y &= 27 \\
   3x + 10y &= -24
   \end{align*} \]
   
   d. \[ \begin{align*}
   \frac{2x + y}{3} &= 15 \\
   \frac{3x - y}{5} &= 1
   \end{align*} \]

29. Reason abstractly. Explain how you would eliminate one of the variables in this system.
   \[ \begin{align*}
   0.4x + 0.5y &= 2.5 \\
   1.2x - 3.5y &= 2.5
   \end{align*} \]

30. Persevere in solving problems. Noah's Cafe sells special blends of coffee mixtures. Noah wants to create a 20-pound mixture with coffee that sells for $9.20 per pound and coffee that sells for $5.50 per pound. How many pounds of each mixture should he blend to create coffee that sells for $6.98 per pound?
   
   A. 8 pounds of the $5.50-per-pound coffee, 12 pounds of the $9.29-per-pound coffee
   
   B. 10 pounds of the $5.50-per-pound coffee, 10 pounds of the $9.29-per-pound coffee
   
   C. 12 pounds of the $5.50-per-pound coffee, 8 pounds of the $9.20-per-pound coffee
   
   D. 15 pounds of the $5.50-per-pound coffee, 5 pounds of the $9.20-per-pound coffee

26. a. \((1, -1)\)
   
   27. a. \((3, 1)\)
      
      b. \((7, 2)\)
      
      c. infinitely many solutions
      
      d. \((2, 5)\)
      
   28. a. \((-4, -7)\)
      
      b. \((-4.5, 3)\)
      
      c. \((-2, -3)\)
      
      d. \((10, 25)\)
      
   29. Sample answer: Multiply the first equation by 3 to get \[ \begin{align*}
   1.2x + 1.5y &= 7.5
   \end{align*} \]. Then subtract the equation \[ \begin{align*}
   1.2x - 3.5y &= 2.5
   \end{align*} \] to eliminate the variable \(x\) and get \(5y = 5\).
   
   30. C
Support for Lesson 3-2

Learning Targets for lesson are found on page 36.

Main Ideas for success in 3-2:

⇒ Solve systems of three linear equations in three variables using Gaussian elimination
⇒ Formulate systems of three linear equations in three variables to model a real-world situation.
⇒ Vocabulary used in this lesson includes ordered triple, Gaussian elimination
⇒ Standards for Mathematical Practice to be demonstrated are make sense of problems, and use appropriate tools strategically

Practice Support for Lesson 3-2

⇒ This example is a model to help solve Practice problem 10.

EXAMPLE:

Solve the system using substitution. \[
\begin{align*}
x + y + z &= 6 \\
2x + y + 2z &= 14 \\
3x + 3y + z &= 8
\end{align*}
\]

Solve the first equations for the variable with a coefficient of one.

\[
\begin{align*}
x + y + z &= 6 \\
z &= -x - y + 6 \\
z &= -x - y + 6
\end{align*}
\]

Substitute the expression for \(z\) into the second equation, and solve for \(y\).

\[
\begin{align*}
2x + y + 2z &= 14 \\
2x + y + 2(-x - y + 6) &= 14 \\
2x + y - 2x - 2y + 12 &= 14 \\
-y &= 2 \\
y &= -2
\end{align*}
\]

Substitute the expression for \(z\) into the third equation, and solve for \(y\).

\[
\begin{align*}
3x + 3y + (-x - y + 6) &= 8 \\
3x + 3y - x - y + 6 &= 8 \\
2x + 2y + 6 &= 8 \\
2x + 2y &= 2
\end{align*}
\]

Plug \(y\) into the previous equation to find the value of \(x\)

\[
\begin{align*}
2x + 2y &= 2 \\
2x + 2(-2) &= 2 \\
2x - 4 &= 2 \\
x &= 3
\end{align*}
\]
Practice Support for Lesson Continued: 3-2

Plug x and y into the first equation to find z.

\[ x + y + z = 6 \]
\[ (3) + (-2) + z = 6 \]
\[ 1 + z = 6 \]
\[ z = 5 \]

Write answer as an ordered pair \((x, y, z)\) \((3, -2, 5)\)

⇒ This example is a model to help solve Practice problems 11.

**EXAMPLE:**

Solve the system using Gaussian elimination.

\[
\begin{align*}
    x - 6y + 4z &= -12 \\
    x + y - 4z &= 12 \\
    2x + 2y + 5z &= -15
\end{align*}
\]

When solving a three variable system using the elimination method, the goal is to get the system to only have two variables. Then proceed to solve the two-variable system using the method of your choice.

Group the first two equations and the second two equations and eliminate the same variable in each group

\[
\begin{align*}
    \{(x - 6y + 4z = -12) \star -1
    \}
    x + y - 4z &= 12 \\
    \{(x + y - 4z = 12) \star -2
    \}
    2x + 2y + 5z &= -15
\end{align*}
\]

For this example eliminate x

\[
\begin{align*}
    \{-(x - 6y + 4z = -12) \star -1
    \}
    (x - 6y - 4z = 12 \\
    \{-(x + y - 4z = 12) \star -2
    \}
    -2x - 2y + 8z = -24 \\
    \{x + y - 4z = 12
    \}
    2x + 2y + 5z = -15
\end{align*}
\]

Add equations together

\[
\begin{align*}
    7y - 8z &= 24 \\
    13z &= -39 \\
    z &= -3
\end{align*}
\]

Group the two resulting equations and solve the two-variable system

\[
\begin{align*}
    \{7y - 8z &= 24 \\
    z &= -3
\end{align*}
\]

Plug z into the top equation in order to find the value of y.

\[
\begin{align*}
    7y - 8(-3) &= 24 \\
    7y + 24 &= 24 \\
    7y &= 0 \\
    y &= 0
\end{align*}
\]
Practice Support for Lesson Continued: 3-2

Plug y and z into one of the first equations in order to find the value of x.

\[ x - 6(0) + 4(-3) = -12 \]
\[ x - 12 = -12 \]
\[ x = 0 \]

Write solution as an ordered pair (x, y, z). (0, 0, -3)

⇒ This example is a model to help solve Practice problem 12.

<table>
<thead>
<tr>
<th>Time</th>
<th>Small cups sold</th>
<th>Med. cups sold</th>
<th>Lg. cups sold</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>48.80</td>
</tr>
<tr>
<td>Afternoon</td>
<td>3</td>
<td>4</td>
<td>1.7</td>
<td>33.60</td>
</tr>
<tr>
<td>Evening</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>49.60</td>
</tr>
</tbody>
</table>

EXAMPLE

Write a system of equations that can be used to determine small, medium, and large, the cost in dollars of each.

Define the variables: x = small cups sold, y = medium cups sold, z = large cups sold

Create three different equations to represent the sales; this will become your system.

Morning sales: \[ 3x + 5y + 4z = 48.80 \]

Afternoon sales: \[ 3x + 4y + 1.7z = 33.60 \]

Evening sales: \[ 5x + 6y + 2z = 49.60 \]

⇒ This example is a model to help solve Practice problem 13.

EXAMPLE

Solve your equation and explain what it means in the context of this situation.

\[ \begin{align*}
3x + 5y + 4z &= 48.80 \\
3x + 4y + 1.7z &= 33.60 \\
5x + 6y + 2z &= 49.60
\end{align*} \]

Choose a method to solve the system. This will model elimination.

Remember to group the first two equations and the last two equations and eliminate the same variable in each group. The goal is to get this three variable system down to a two variable system so it is easier to solve.

\[ \begin{align*}
(3x + 5y + 4z = 48.80) &\times -1 \\
3x + 4y + 1.7z = 33.60
\end{align*} \]
\[ \begin{align*}
(3x + 4y + 1.7z = 33.60) &\times 5 \\
(5x + 6y + 2z = 49.60) &\times 3
\end{align*} \]

\[ \begin{align*}
-3x - 5y - 4z &= -48.80 \\
3x + 4y + 1.7z &= 33.60 \\
15x + 20y + 8.5z &= 168 \\
-15x - 18y - 6z &= 148.80
\end{align*} \]

\[ \begin{align*}
-y - 2.3z &= -15.20 \\
2y + 2.5z &= 19.20
\end{align*} \]

\[ \begin{align*}
-y - 2.3z &= -15.20 \\
2y + 2.5z &= 19.20
\end{align*} \]
Choose a method to solve the two variable system

\[
\begin{align*}
-y - 2.3z &= -15.20 \\
2y + 2.5z &= 19.20
\end{align*}
\]

\[
\begin{align*}
(-y - 2.3z = -15.20) \times 2 \\
2y + 2.5z &= 19.20
\end{align*}
\]

\[
\begin{align*}
2y - 4.6z &= -30.40 \\
2y + 2.5z &= 19.20
\end{align*}
\]

\[
\begin{align*}
-2.1z &= -11.20 \\
z &= 5.33
\end{align*}
\]

Plug \( z \) into one of the two-variable equations in order to find the value of \( y \).

\[
2y + 2.5(5.33) = 19.20
\]

\[
2y + 13.33 = 19.20
\]

\[
2y = 5.87
\]

\[
y = 2.94
\]

Plug \( y \) and \( z \) into one of the three-variable equations to find the value of \( x \).

\[
3x + 5y + 4z = 48.80
\]

\[
3x + 5(2.94) + 4(5.33) = 48.80
\]

\[
3x + 14.70 + 21.32 = 48.80
\]

\[
3x + 36.02 = 48.80
\]

\[
3x = 12.78
\]

\[
x = 4.26
\]

The solution to this system is \((4.26, 2.94, 5.33)\). In this context this solution means that a small cup sold for $4.26, a medium cup sold for $2.94, and a large cup sold for $5.33.

\[\Rightarrow\] This example is a model to help solve Practice problem 14.

**EXAMPLE**

Which method did you use to solve the system? Explain why you used that method.

Answers here may vary. In the example, elimination was used in order to reduce the three-variable system to a two-variable system so it was easier to solve.
31. Solve each system by substitution.
   a. \[ 3x + y + 4z = -10 \\
       -x + y + 2z = 6 \\
       2x - y + z = -8 \]
   b. \[ x - 6y - 2z = -8 \\
       -x + 5y + 3z = 2 \\
       3x - 3y - 4z = 15 \]
   c. \[ x + 3y - z = 7 \\
       2x - y + 2z = 3 \\
       3x + 2y - z = 2 \]
   d. \[ 3x + y - z = 9 \\
       x + 2y + 3z = 8 \\
       2x + 3y + z = 16 \]

32. Solve each system using Gaussian elimination.
   a. \[ 4x + y - 3z = 11 \\
       2x - 3y + 2z = 9 \\
       x + y + z = -3 \]
   b. \[ 5x - 3y + 2z = -8 \\
       2x + 4y - z = 18 \\
       x - 6y + 4z = -25 \]
   c. \[ x - 3y + z = 1 \\
       3x - 5y + 2z = -4 \\
       x - 2y - 3z = -20 \]
   d. \[ 2x + 4y + z = 3 \\
       x - y - 2z = 2 \\
       x + 2y - 3z = -1 \]

33. Use the table for parts a and b.

<table>
<thead>
<tr>
<th>Smoothie Sales</th>
<th>Time Period</th>
<th>Classic No. of cups sold</th>
<th>Fruit No. of cups sold</th>
<th>Veggie No. of cups sold</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8:00 - 12:00</td>
<td>10</td>
<td>12</td>
<td>3</td>
<td>$103.50</td>
</tr>
<tr>
<td></td>
<td>12:00 - 4:00</td>
<td>6</td>
<td>8</td>
<td>15</td>
<td>$125.50</td>
</tr>
<tr>
<td></td>
<td>4:00 - 8:00</td>
<td>12</td>
<td>6</td>
<td>8</td>
<td>$102.00</td>
</tr>
</tbody>
</table>

   a. Write a system of equations that can be used to determine \( c, f, \) and \( v, \) the price in dollars of a classic, fruit, and veggie smoothie.

   b. Solve your system and explain what the solution means in the context of the situation.

34. A food store makes a 10-pound mixture of peanuts, cashews, and raisins. The mixture has twice as many peanuts as cashews. Peanuts cost $1 per pound, cashews cost $2 per pound, and raisins cost $2 per pound. The total cost of the mixture is $16. How much of each ingredient did the store use to make the mixture?

35. Make sense of problems. A company placed $500,000 in three different investment plans. The company placed some money into short-term notes paying 5.5% per year. They placed three times as much into government bonds paying 5% per year. They placed the rest in utility bonds paying 4.5% per year. The income for one year was $25,000. How much more money did the company place in government bonds than in utility bonds?
   A. $100,000
   B. $200,000
   C. $300,000
   D. $400,000
Additional Practice Answers:

31. a. $(-2, 4, 0)$
   b. $(4, 3, -3)$
   c. $\left(\frac{4}{7}, \frac{27}{7}, 4\right)$
   d. $(1, 5, -1)$

32. a. $(2, -3, -2)$
   b. $\left(1, \frac{19}{5}, -\frac{4}{5}\right)$
   c. $(-3, 1, 5)$
   d. no solution

33. a. \[
\begin{align*}
10c + 12f + 3v &= 103.50 \\
6c + 8f + 15v &= 125.50 \\
12c + 6f + 8v &= 102.00
\end{align*}
\]
   b. $(3, 5, 4.5)$; A classic is $3$, a fruit is $5$, and a veggie is $4.50$.

34. 4 pounds of peanuts, 2 pounds of cashews, and 4 pounds of raisins

35. B
Support for Lesson 3-3

**Learning Targets** for lesson are found on page 42.

**Main Ideas for success in 3-3:**

⇒ Add, subtract, and multiply matrices.
⇒ Use a graphing calculator to perform operations on matrices.
⇒ Vocabulary used in this lesson includes: matrix, dimensions of a matrix, entries
⇒ Standards for Mathematical Practice to be demonstrated are: Make Use Of Structure, Express Regularity In Repeated Reasoning, Critique The Reasoning Of Others

**Practice Support for Lesson 3-3**

⇒ This example is a model to help solve Practice problem 12.

**EXAMPLE:** Use these matrices to answer items 12-17. What are the dimensions of \( A \)?

\[
A = \begin{pmatrix}
4 & -1 & 5 \\
-3 & 0 & 7
\end{pmatrix} \\
B = \begin{pmatrix}
3 & -2 \\
1 & 8
\end{pmatrix} \\
C = \begin{pmatrix}
9 & -4 \\
3 & -1
\end{pmatrix}
\]

2 x 3  number of rows x number of columns

⇒ This example is a model to help solve Practice problem 13.

**EXAMPLE:** Use these matrices from above, what is the entry with the address \( C_{21} \)?

3. Look at matrix \( C \) for the entry on the second row and the first column

⇒ This example is a model to help solve Practice problem 14.

**EXAMPLE:** Use these matrices from above, find \( B + C \).

\[
B + C = \begin{pmatrix}
12 & -6 \\
4 & 7
\end{pmatrix}
\]

Both matrices have the same dimensions, they can be added together. Add the entries that share the same address.

\( B_{11} = 3 \) and \( C_{11} = 9 \)  \( B_{11} + C_{11} = 3 + 9 = 12 \)

⇒ This example is a model to help solve Practice problem 15.

**EXAMPLE:** Use these matrices from above, find \( C - B \).

\[
C - B = \begin{pmatrix}
6 & -2 \\
2 & -9
\end{pmatrix}
\]

Both matrices have the same dimensions, they can be subtracted together. Subtract the entries that share the same address. \( C_{11} = 9 \) and \( B_{11} = 3 \)  \( C_{11} - B_{11} = 9 - 3 = 6 \)
This example is a model to help solve Practice problem 16.

**EXAMPLE:** Use these matrices from above, find $AB$ if it is defined.

The dimensions of $A$ are $2 \times 3$ and the dimensions of $B$ are $2 \times 2$. In order to multiply matrices, the number of columns in $A$ is equal to the number of rows in $B$. Since there are 3 columns in $A$ and only 2 rows in $B$, we cannot multiply the matrices $A$ and $B$. Therefore it is not defined.

This example is a model to help solve Practice problem 17.

**EXAMPLE:** Use these matrices from above, find $BC$ if it is defined.

The dimensions of $B$ are $2 \times 2$ and the dimensions of $C$ are $2 \times 2$. Since there are 2 columns in $B$ and 2 rows in $C$, we can multiply the matrices $B$ and $C$ because it is defined.

Find the entry in row 1, column 1 of $BC$. Use row 1 of $B$ and column 1 of $C$. Multiply the first entries and the second entries. Then add the products. $3(9) + (-2)(3) = 21$

$$BC = \begin{bmatrix} 3 & -2 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 9 & -4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 21 \\ 9 \end{bmatrix}$$

Find the entry in row 1, column 2 of $BC$. Use row 1 of $B$ and column 2 of $C$. Multiply the first entries and the second entries. Then add the products. $3(-4) + (-2)(-1) = -9$

$$BC = \begin{bmatrix} 3 & -2 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 9 & -4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 21 & -9 \end{bmatrix}$$

Find the entry in row 2, column 1 of $BC$. Use row 2 of $B$ and column 1 of $C$. Multiply the first entries and the second entries. Then add the products. $1(9) + 8(3) = 33$

$$BC = \begin{bmatrix} 3 & -2 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 9 & -4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 21 \\ 33 \end{bmatrix}$$

Find the entry in row 2, column 2 of $BC$. Use row 2 of $B$ and column 2 of $C$. Multiply the first entries and the second entries. Then add the products. $1(-4) + 8(-1) = -12$

$$BC = \begin{bmatrix} 3 & -2 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 9 & -4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 21 & -9 \\ 33 & -12 \end{bmatrix}$$

Solution: $BC = \begin{bmatrix} 21 & -9 \\ 33 & -12 \end{bmatrix}$
Additional Practice:

1. Which entry corresponds to the address $c_{32}$ for the following matrix?

$$C = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$$

A. 3  
B. 4  
C. -1  
D. 0

2. A is a $3 \times 2$ matrix. B is a $2 \times 3$ matrix. What are the dimensions of the matrix product, $AB$?

3. Find the sum or difference.

$$A = \begin{bmatrix} 2 & -1 & 8 \\ 3 & 1 & -1 \\ 2 & 1 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & -5 \\ 4 & 2 & 7 \\ -3 & 9 & 12 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 6 & 14 \\ 10 & 8 & -7 \\ 0 & -4 & -9 \end{bmatrix}$$

a. $A + B$  
b. $A - B$  
c. $B + C$  
d. $C - B$

Answers to additional practice:

1. B
2. $3 \times 3$
3. 
   a. $\begin{bmatrix} 3 & 2 & 3 \\ 7 & 3 & 6 \\ -1 & 10 & 18 \end{bmatrix}$
   b. $\begin{bmatrix} 1 & -4 & 13 \\ -1 & -1 & -8 \\ 5 & -8 & -4 \end{bmatrix}$
   c. $\begin{bmatrix} 4 & 9 & 9 \\ 14 & 10 & 0 \\ -3 & 5 & 3 \end{bmatrix}$
   d. $\begin{bmatrix} 2 & 3 & 19 \\ 6 & 6 & -14 \\ 3 & -13 & -21 \end{bmatrix}$
Additional Practice:

4. Find the product.

\[
D = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} \quad E = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}
\]

\[
F = \begin{bmatrix} -1 & 3 & 5 \\ 4 & -2 & 0 \end{bmatrix}
\]

a. \(DE\)  \quad b. \(DF\)

c. \(EF\)  \quad d. \(FD\)

5. **Make use of structure.** Use the following matrices for parts a and b.

\[
G = \begin{bmatrix} 2 & -4 \\ -1 & 2 \\ 5 & 3 \end{bmatrix} \quad H = \begin{bmatrix} -3 & 1 & 6 \\ 2 & 5 & -2 \end{bmatrix}
\]

a. Can you add or subtract the matrices? Explain.

b. Can you multiply the matrices, \(GH\)? Explain.

Answers to additional practice:

4. a. \[
\begin{bmatrix} -9 & -3 \\ -16 & 2 \end{bmatrix}
\]

b. \[
\begin{bmatrix} 17 & -1 & 15 \\ 18 & -14 & -10 \end{bmatrix}
\]

c. \[
\begin{bmatrix} -6 & 8 & 10 \\ 3 & -9 & -15 \end{bmatrix}
\]

d. not defined

5. a. No. The matrices are not the same dimensions.

b. Yes. The number of rows in \(G\) is the same as the number of columns in \(H\).
Support for Lesson 3-4

**Learning Targets** for lesson are found on page 47.

**Main Ideas for success in Lesson 3-4:**

⇒ Solve systems of two linear equations in two variables by using graphing calculators with matrices.
⇒ Solve systems of three linear equations in three variables by using graphing calculators with matrices.
⇒ Vocabulary used in this lesson includes: square matrix, multiplicative identity matrix, multiplicative inverse matrix, matrix equation, coefficient matrix, variable matrix, constant matrix
⇒ Standards for Mathematical Practice to be demonstrated are: Construct Viable Arguments, Make Sense Of Problems, Model With Mathematics, Use Appropriate Tools Strategically, Critique The Reasoning Of Others.

**Practice Support for Lesson 3-4**

⇒ This example is a model to help solve Practice problem 19.

**EXAMPLE:** Use a graphing calculator to find the inverse of the matrix \[
\begin{bmatrix}
4 & 8 \\
5 & 2
\end{bmatrix}
\].

Input the matrix into the graphing calculator. 2nd → MATRIX → EDIT → ENTER → 2X2 → ENTER → 4 → ENTER → 8 → ENTER → 5 → ENTER → 2 → ENTER → 2nd → QUIT → 2nd → MATRIX → ENTER → x⁻¹ → ENTER

\[
\begin{bmatrix}
-0.625 & 0.25 \\
1.5625 & -0.125
\end{bmatrix}
\]

This is the inverse of the given matrix. To change from decimals to fractions press MATH → ENTER → ENTER to get

\[
\begin{bmatrix}
\frac{1}{16} & \frac{1}{4} \\
\frac{5}{32} & \frac{1}{8}
\end{bmatrix}
\]

⇒ This example is a model to help solve Practice problems 20-21.

**EXAMPLE:** Write a matrix equation to model the system. Then use the matrix equation to solve the system.

\[
\begin{align*}
2x + y - z &= 4 \\
-2x + y + 2z &= 6 \\
x + 2y + z &= 11
\end{align*}
\]

First write the system as a matrix equation. \[
\begin{align*}
A \cdot X &= B
\end{align*}
\]

\[
\begin{bmatrix}
2 & 1 & -1 \\
-2 & 1 & 2 \\
1 & 2 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\begin{bmatrix}
4 \\
6 \\
11
\end{bmatrix}
\]

Matrix A represents the coefficients in each equation, Matrix X represents the variables in each equation, Matrix B represents the constants in each equation.

To solve the matrix equation, we can multiply both sides of the equation by the multiplicative inverse matrix of A. This will give us \(X = A^{-1}B\). Use a graphing calculator to find \(A^{-1}\) as you did in the earlier problem.

\[
A^{-1} = \begin{bmatrix}
\frac{1}{4} & -\frac{1}{3} & \frac{1}{3} \\
\frac{5}{3} & 1 & -\frac{2}{3} \\
-\frac{5}{3} & -1 & \frac{4}{3}
\end{bmatrix}
\]

Now you can find the matrix product of \(A^{-1}B\).

\[
\begin{bmatrix}
\frac{1}{4} & -\frac{1}{3} & \frac{1}{3} \\
\frac{5}{3} & 1 & -\frac{2}{3} \\
-\frac{5}{3} & -1 & \frac{4}{3}
\end{bmatrix}
\begin{bmatrix}
4 \\
6 \\
11
\end{bmatrix}
= \begin{bmatrix}
1 \\
4 \\
2
\end{bmatrix}
\]

(Use the graphing calculator to multiply)
EXAMPLE David has 3 euros and 9 British pound worth a total of $75. Daniel has 8 euros and 5 British pounds worth a total of $67.

a.) Write a system of equations to model this situation, where $x$ represents the value of 1 euro in dollars and $y$ represents the value of 1 British pound in dollars.

\[
\begin{align*}
3x + 9y & = 75 \\
18x + 5y & = 67
\end{align*}
\]

b.) Write the system of equations as a matrix equation.

\[
\begin{bmatrix}
3 & 9 \\
8 & 5
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
75 \\
67
\end{bmatrix}
\]

c.) Use the matrix equation to solve the system. Then interpret the solution.

As in the earlier examples, use the graphing calculator to input matrix A (the coefficient matrix) and matrix B (the constant matrix).

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix} 3 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 75 \\
67
\end{bmatrix} \quad \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix} 4 \\
7
\end{bmatrix}
\]

(4, 7); A euro is worth $4, and a British pound is worth $7.
Additional Practice:

1. Which of the following is the inverse of \[
\begin{pmatrix}
2 & 1 \\
0 & 3
\end{pmatrix}.
\]
   A. \[
\begin{pmatrix}
\frac{1}{3} & \frac{1}{2} \\
-\frac{1}{2} & \frac{1}{6}
\end{pmatrix}
\]  
   B. \[
\begin{pmatrix}
\frac{1}{2} & \frac{1}{6} \\
0 & \frac{1}{3}
\end{pmatrix}
\]
   C. \[
\begin{pmatrix}
\frac{1}{3} & -\frac{1}{2} \\
0 & \frac{1}{2}
\end{pmatrix}
\]  
   D. \[
\begin{pmatrix}
\frac{1}{2} & \frac{1}{6} \\
0 & -\frac{1}{3}
\end{pmatrix}
\]

2. **Model with mathematics.** Write a matrix equation to model each system. Then use the matrix equation to solve the system.
   a. \[
\begin{align*}
-x + y &= 20 \\
-2x + y &= 10
\end{align*}
\]
   \[
\begin{pmatrix}
2x + 3y + z &= 4 \\
3x + 3y + z &= 8 \\
2x + 4y + z &= 5
\end{pmatrix}
\]

3. Lori bought cheeses for a party. The cheddar cost $7.95 per pound, and the Swiss cost $9.50 per pound. Lori bought \(\frac{5}{2}\) pounds of cheese for a total of $47.60.
   a. Write a system of equations that can be used to determine how many pounds of each type of cheese Lori bought. Let \(x\) be the number of pounds of cheddar cheese, and let \(y\) be the number of pounds of Swiss cheese.
   b. Write a matrix equation that can be used to solve the system.
   c. Use the matrix equation to solve the system.
   d. Explain the meaning of the solution.

4. The local bookstore had a sale. The table shows the number of items sold over three days.

<table>
<thead>
<tr>
<th>Day</th>
<th>Paperback</th>
<th>Hardcover</th>
<th>e-book</th>
<th>Amount of Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>55</td>
<td>17</td>
<td>33</td>
<td>$970</td>
</tr>
<tr>
<td>Tuesday</td>
<td>12</td>
<td>10</td>
<td>13</td>
<td>$335</td>
</tr>
<tr>
<td>Wednesday</td>
<td>23</td>
<td>18</td>
<td>19</td>
<td>$595</td>
</tr>
</tbody>
</table>

a. Write a matrix equation that can be used to find the price of each item.

b. Solve the equation to find the solution of the system and explain the meaning of the solution.

5. Mari has 42 coins consisting of nickels, dimes, and quarters. The total value of the coins is $9.35. The number of dimes is one less than three times the number of nickels. How many of each type of coin does Mari have?

**Answers to Additional Practice:**

\[
\begin{pmatrix}
1 \\
-2
\end{pmatrix} = 
\begin{pmatrix}
x \\
y
\end{pmatrix} = 
\begin{pmatrix}
20 \\
10
\end{pmatrix}
\]

\[
\begin{pmatrix}
2 & 3 & 1 \\
3 & 1 & 1 \\
2 & 4 & 1
\end{pmatrix} 
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = 
\begin{pmatrix}
4 \\
8 \\
5
\end{pmatrix}
\]

\[
\begin{align*}
x + y &= 5.5 \\
7.95x + 9.5y &= 47.6
\end{align*}
\]

\[
\begin{pmatrix}
1 & 1 \\
7.95 & 9.5
\end{pmatrix} 
\begin{pmatrix}
x \\
y
\end{pmatrix} = 
\begin{pmatrix}
5.5 \\
47.6
\end{pmatrix}
\]

Lori bought 3 pounds of cheddar and 2.5 or \(\frac{5}{2}\) pounds of Swiss for the party.

\[
\begin{pmatrix}
55 \\
17 \\
12 \\
10 \\
13 \\
23 \\
18 \\
19
\end{pmatrix} 
\begin{pmatrix}
p \\
h
\end{pmatrix} = 
\begin{pmatrix}
970 \\
335 \\
595
\end{pmatrix}
\]

During the sale, paperback books cost $10, hardcover books cost $15, and e-books cost $5.

Marie has 2 nickels, 5 dimes, and 35 quarters.
Support for Lesson 4-1

**Learning Targets** for lesson are found on page 57.

**Main Ideas for success in Lesson 4-1:**

⇒ Graph piecewise-defined functions.

⇒ Write the domain and range of functions using interval notation, inequalities, and set notation.

⇒ Vocabulary used in this lesson includes: piecewise-defined function, domain, range.

⇒ Standards for Mathematical Practice to be demonstrated are: Make Use Of Structure, Model With Mathematics, Critique The Reasoning Of Others.

**Practice Support for Lesson 4-1**

⇒ This example is a model to help solve Practice problem 14.

**EXAMPLE:** Graph the piecewise-defined function. Then write its domain and range using inequalities, interval notation, and set notation.

\[ f(x) = \begin{cases} 
-2x + 3 & \text{if } x < 0 \\
 x^2 & \text{if } x \geq 0 
\end{cases} \]

The domain refers to all defined x-values in the function. The function is defined for all x-values from \(-\infty \) to \(\infty\).

Domain: Inequalities \(-\infty < x < \infty\), Interval Notation \((-\infty, \infty)\), Set Notation \(\{x | x \in \mathbb{R}\}\)  

The range refers to all defined y-values in the function. The function is defined for all y-values that are 0 and greater.

Range: Inequalities \(y \geq 0\), Interval Notation \([0, \infty)\), Set Notation \(\{y | y \in \mathbb{R}, y \geq 0\}\)  

⇒ This example is a model to help solve Practice problem 15.

**EXAMPLE:** The range of a function is all real numbers greater than or equal to -12 and less than or equal to 3. Write the range of the function using and inequality, interval notation, and set notation.

Range: Inequalities \(-12 \leq y \leq 3\), Interval Notation \([-12, 3]\), Set Notation \(\{y | y \in \mathbb{R}, -12 \leq y \leq 3\}\)  

Since the description says “or equal to” we are sure to use brackets, not parentheses, and include the ‘or equal to’ part under our inequality signs.
This example is a model to help solve Practice problem 16.

**EXAMPLE:** Evaluate the piecewise function for \( x = -1, x = 0, \text{ and } x = 2 \).

\[
f(x) = \begin{cases} 
9x + 6 & \text{if } x < -1 \\
x^2 & \text{if } -1 \leq x < 2 \\
x - 5 & \text{if } x \geq 2 
\end{cases}
\]

To evaluate \( x = -1 \) we must first figure out where it is defined (which of the 3 parts has a domain that includes -1). We see it is included in the middle piece. We can now plug in -1 for x and evaluate to get \((-1)^2 = 1\).

Similarly, \( x = 0 \) is defined for the middle piece. We can then evaluate \((0)^2 = 0\).

Lastly, \( x = 2 \) is defined for the third piece of the function, when evaluated, \(2 - 5 = -3\).

Note that the first piece of the function was not used at all in this case, that is okay.

This example is a model to help solve Practice problem 17.

**EXAMPLE** A cell phone company charges customers a $15 fee plus $0.10 per text for the first 1,000 texts and $0.20 per text for each text sent after the 1000th text.

a.) Write a piecewise function \( f(x) \) that can be used to determine a customer’s monthly bill for using \( x \) texts per month.

This function will have two pieces to it since there are two possible pricing scenarios. Either the customer sent fewer than (or exactly) 1,000 texts, or more than 1,000 texts in the month.

\[
f(x) = \begin{cases} 
15 + 0.10x & \text{if } 0 \leq x \leq 1,000 \\
115 + 0.20(x - 1,000) & \text{if } x > 1,000 
\end{cases}
\]

The first piece, \(15 + 0.10x\), comes from the initial $15 fee applied monthly to everyone plus the charge of $0.10 per text. This piece is only used when the total number of monthly texts is 1,000 or fewer.

The second piece, \(115 + 0.20(x - 1,000)\), comes from the scenario when the customer sends more than 1,000 texts in a given month. The $115 represents the total charge for the initial $15 fee applied to all customers monthly plus the $100 they are charged for sending their first 1,000 texts. The second half represents the cost charged for the additional texts (after the first 1,000). Take the total number of texts sent, \(x\), and subtract 1,000 (since we are already paying for those in our $115). The number of texts left over after the subtraction is the amount that will be charged at the $0.20 rate.

b.) Graph the piecewise function.

c.) A customer sends 1234 texts in one month. How much will the customer owe the cell phone company? Explain how you determined your answer.

Since 1234 is more than 1,000, I need to evaluate in the bottom piece of my function. \(115 + 0.20(1234 - 1,000) = 161.8\). The customer will owe the company $161.80.
Additional Practice:

1. The domain of a function is all real numbers greater than or equal to $-3$ and less than 12. Write the domain using an inequality, interval notation, and set notation.

2. The range of a function is $(-\infty, 5]$. Which of the following shows this range using set notation?
   - A. $y \leq 5$
   - B. $\{y \mid y \in \mathbb{R}, y \leq 5\}$
   - C. $y < 5$
   - D. $\{y \mid y \in \mathbb{R}, y < 5\}$

3. Graph each piecewise function. Then write its domain and range using inequalities, interval notation, and set notation.
   \[ f(x) = \begin{cases} 
   2x & \text{if } x \leq 2 \\
   -\frac{1}{2}x + 2 & \text{if } x > 2 
   \end{cases} \]

4. Evaluate the piecewise function for $x = -3$, $x = 3$, and $x = 30$.

5. Model with mathematics. A water company charges residential customers a quarterly flat fee of $25 for up to 5000 gallons of water. The company charges $10 per thousand for the first 10,000 gallons of water after the minimum allowance. The next 10,000 gallons is $5 per thousand. All other consumption over 25,000 gallons is $2.50 per thousand.

   a. Write a piecewise function $w(x)$ that can be used to determine a customer's quarterly bill for using $x$ gallons of water.

   b. Graph the piecewise function.

   c. Why are the slopes of each piece of the function different? Explain.

   d. On average, a family of four consumes 247.2 gallons of water per day. Calculate the quarterly water bill for a family of four. Explain how you determined your answer.

Answers to Additional Practice:

1. $-3 \leq x < 12$, $\{x \mid x \in \mathbb{R}, -3 \leq x < 12\}
   \hspace{1cm} D$

2. $\{y \mid y \in \mathbb{R}, -\infty < y \leq 2\}$

3. $h(-3) = -11$, $h(30)$ is undefined, $h(30) = 895$

4. $w(x) = \begin{cases} 
   25 & \text{if } 0 < x \leq 5000 \\
   0.01x + 50 & \text{if } 5000 < x \leq 15,000 \\
   0.005x + 112.5 & \text{if } 15,000 < x \leq 25,000 \\
   0.0025x + 112.5 & \text{if } x > 25,000 
   \end{cases} \]

   a. Water Charges

   b. Cost ($\) vs. Water (thousands of gallons)

   c. Since the rate or charge for the water changes with the amount of consumption, the slope for each piece of the function is different.

   d. $162.79; \text{ first determine the water consumption for a quarter of the year. Since there are 91.25 days in a quarter, the water consumption for a quarter is 247.2 gallons per day } \times 91.25 \text{ days or 22,557 gallons. Next use the part of the piecewise function that corresponds to the range for the given number of gallons. Substituting 22,557 gallons for } x \text{ in } 0.0025x + 112.5 = 162.79.$
Support for Lesson 4-2

Learning Targets for lesson are found on page 61.

Main Ideas for success in Lesson 4-2:

⇒ Graph step functions and absolute value functions.
⇒ Describe the attributes of these functions.
⇒ Vocabulary used in this lesson includes: step function
⇒ Standards for Mathematical Practice to be demonstrated are: Reason Abstractly, Make Sense Of Problems, Reason Quantitatively, Construct Viable Arguments, Make Sense Of Problems And Persevere In Solving Them

Practice Support for Lesson 4-2

⇒ This example is a model to help solve Practice problem 13.

EXAMPLE: A step function known as the ceiling function, written \( g(x) = \lceil x \rceil \), yields the value \( g(x) \) that is the least integer greater than or equal to \( x \).

a.) Graph this step function. (See to the right)

b.) Find \( g(\frac{17}{10}) \) and \( g(-\frac{17}{10}) \).

The smallest integer that is larger than \( 1.7 \) is 2. Therefore \( g(1.7) = 2 \).

Similarly, the smallest integer that is larger than \( -\frac{17}{10} \) is -2. Therefore \( g(-\frac{17}{10}) = -2 \).

⇒ This example is a model to help solve Practice problem 14.

EXAMPLE: Use the information below to aid in answering questions 14 and 15.

A day ticket for admission to a water park is $45 for children at least 5 years old and less than 13 years old. A day ticket for children at least 13 years old and less than 18 years old costs $55. A day ticket for adults, anyone 18 years or older, costs $65.

Write the equation of a step function \( f(x) \) that can be used to determine the cost in dollars of a day ticket for the water park for a person who is \( x \) years old.

\[
f(x) = \begin{cases} 
45 & \text{if } 5 \leq x < 13 \\
55 & \text{if } 13 \leq x < 18 \\
65 & \text{if } x \geq 18 
\end{cases}
\]

⇒ This example is a model to help solve Practice problem 15.

EXAMPLE: Graph the step function you wrote above.
Practice Support for Lesson 4-2 Continued:

This example is a model to help solve Practice problem 16.

**EXAMPLE** Use the absolute value function \( h(x) = |x - 4| \) for questions 16-19.

Graph the absolute value function.

This example is a model to help solve Practice problem 17.

**EXAMPLE:** What are the domain and range of the function?

Domain: \( \{x | x \in \mathbb{R}\} \); Range: \( \{y | y \in \mathbb{R}, y \geq 0\} \)

This example is a model to help solve Practice problem 18.

**EXAMPLE** What are the coordinates of the vertex of the function’s graph?

\((4, 0)\) It is the parent function shifted to the right 4 units.

This example is a model to help solve Practice problem 19.

**EXAMPLE:** Write the equation for the function using piecewise notation.

\[
h(x) = \begin{cases} 
-x + 4 & \text{if } x < 4 \\
x - 4 & \text{if } x \geq 4
\end{cases}
\]
### Additional Practice:

1. A step function, known as the greatest integer function or the floor function, is written as \( f(x) = \lfloor x \rfloor \), the largest integer not greater than \( x \).
   a. Graph this step function.
   
   \[
   \text{Floor function } f(x) = \lfloor x \rfloor
   \]

   b. Find \( f(2.4), f(0.05), \) and \( f(-3.6) \).

2. Use the absolute value function \( h(x) = |x - 3| \) for parts a–d.
   a. Graph \( h(x) \).

   b. What are the domain and range of the function?

   c. What are the coordinates of the vertex of the function's graph?

   d. Write the equation for the function using piecewise notation.

3. **Model with mathematics.** A tutor charges $45 for each hour or for any fraction of an hour.
   a. Write a function \( t(x) \) to represent this situation using the ceiling function \( \lceil x \rceil \), the smallest integer greater than or equal to \( x \).

### Answers to Additional Practice:

1. a. 

   ![Graph of \( f(x) \)]

   b. \( f(2.4) = 2, f(0.05) = 0, f(-3.6) = -4 \)

2. a. 

   ![Graph of \( h(x) \)]

   b. domain: \( \{x | x \in \mathbb{R}\} \), range: \( \{y | y \in \mathbb{R}, y = 0\} \)

   c. \( (3,0) \)

   d. \( f(x) = \begin{cases} x + 3 & \text{if } x < 3 \\ x - 3 & \text{if } x \geq 3 \end{cases} \)

3. a. \( t(x) = 45 \lceil x \rceil \)

   b. 

   ![Graph of \( t(x) \)]

   c. $90, $180

   d. B
Additional Practice:

4. Use the step function \( h(x) \) for parts a and b.

\[
h(x) = \begin{cases} 
0 & \text{if } 0 < x < 1 \\
2 & \text{if } 1 < x < 3 \\
1 & \text{if } x > 3 
\end{cases}
\]

a. Graph the function.

[Graph of the function]

b. Evaluate the function for \( x = 0.3, x = 3, x = 1.1, \) and \( x = 100. \)

5. Maura gets $8 per hour for the first 40 hours she works in a week. She gets 1.5 times her hourly rate for every hour she works over 40 hours per week. Maura worked 46 hours last week. How much did Maura earn last week? Is this a step function? Explain.

Answers to Additional Practice:

4. a. [Graph of the function]

b. \( h(0.3) = 0, h(3) = 2, h(1.1) = 1, h(100) = 3 \)

5. A step function is a piecewise function with a constant value throughout each interval of its domain. The value of this function changes with each hour. It is a piecewise function because there are two different rates based on the number of hours worked.
Support for Lesson 4-3

Learning Targets for lesson are found on page 65.

Main Ideas for success in:

⇒ Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \cdot f(x), f(kx), \text{and } f(x + k) \).
⇒ Find the value of \( k \), given these graphs.
⇒ Vocabulary used in this lesson includes: vertical/horizontal translation, reflection, vertical/horizontal stretch, shrink
⇒ Standards for Mathematical Practice to be demonstrated are: Model With Mathematics, Reason Abstractly And Quantitatively, Express Regularity In Repeated Reasoning, Attend To Precision

Practice Support for Lesson

⇒ This example is a model to help solve Practice problem 15.

**EXAMPLE**: The graph of \( g(x) \) is the graph of \( f(x) = |x| \) translated 4 units to the left, write the equation of \( g(x) \).

Since translating left and right appears in the equation on the inside of the absolute value bars and \( |x + k| \) where \( k > 0 \) shifts the graph \( k \) units to the left, our equation must be \( g(x) = |x + 4| \).

⇒ This example is a model to help solve Practice problem 16.

**EXAMPLE**: Describe the graph of \( h(x) = -3|x| \) as one or more transformations of the graph of \( f(x) = |x| \).

A negative sign in the front of the equation means that the graph is reflected across the \( x \)-axis. The 3 in the front of the equation in front of the absolute value bars will cause the function to be vertically stretched by a factor of 3.

⇒ This example is a model to help solve Practice problem 17.

**EXAMPLE**: What are the domain and range of \( f(x) = |x - 2| + 3 \)? Explain.

Since the two transformations applied to the parent function are a horizontal shift right 2 units and a vertical shift up 3 units. The graph will open upward and have a vertex of \((2, 3)\). Domain of the parent function would be \( \{x \in \mathbb{R} \} \) the domain of \( f(x) \) remains the same it is still defined for all values of \( x \). Range of the parent function would be \( \{y \in \mathbb{R} \mid y \geq 0 \} \) however since we have shifted our function up 3 units it is no longer defined from \( 0 \leq y < 3 \) and the new range becomes \( \{y \in \mathbb{R} \mid y \geq 3 \} \).

⇒ This example is a model to help solve Practice problem 18.

**EXAMPLE**: Graph each transformation.

a.) \( r(x) = -|x + 3| - 2 \)  
   b.) \( t(x) = |2x| - 1 \)  
   c.) \( w(x) = -4|x - 2| + 3 \)

Reflected across the \( x \)-axis.  
Horizontally shrink by a factor of \( \frac{1}{2} \)  
Reflected across the \( x \)-axis.  
Horizontal translation left 3  
Vertical translation down 1  
Horizontal translation right 2  
Vertical stretch by a factor of 4  
Vertical translation up 3
EXAMPLE: Write the equation for each transformation of $f(x) = |x|$ described below.

a.) Translate right 7 units, stretch vertically by a factor of 3, and translate up 17 units.

$$a(x) = 3|x - 7| + 17$$

b.) Translate left 3 units, stretch horizontally by a factor of 8, and reflect over the x-axis.

$$b(x) = -\frac{1}{8}(x + 3)$$

c.)

$$c(x) = -2|x - 6| + 4$$
Additional Practice:

1. Write the equation \( g(x) \) of each transformation of the parent graph \( f(x) = |x| \) described below.
   a. reflection over the \( z \)-axis
   b. stretched horizontally by a factor of 3
   c. translated right 5 units
   d. stretched vertically by a factor of 4, translated up 2 units
   e. stretched vertically by a factor of 2, reflected over the \( x \)-axis, translated left 1 unit and down 3 units

2. What is the domain and range of \( g(x) = -|x + 2| - 1 \)?

3. Write the equation for the transformation of \( f(x) = |x| \) as shown in the graph.

![Graph of \( f(x) = |x| \)](image)

4. **Make use of structure.** Find the coordinates of the vertex of \( g(x) = |x + 1| - 2 \).
   A. \((-2, -1)\)  
   B. \((-2, 1)\)  
   C. \((-1, -2)\)  
   D. \((1, -2)\)

5. Graph \( g(x) = \frac{1}{2}|x + 2| - 3 \) as a transformation of \( f(x) = |x| \). Then identify the transformation.

![Graph of \( g(x) = \frac{1}{2}|x + 2| - 3 \)](image)

Answers to Additional Practice:

<p>| | |</p>
<table>
<thead>
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| 1 | a. \( g(x) = -|x| \)  
   b. \( g(x) = \frac{1}{3}|x| \)  
   c. \( g(x) = |x - 5| \)  
   d. \( g(x) = 4|x| + 2 \)  
   e. \( g(x) = -2|x + 1| - 3 \)  
   domain: \( \{x \in \mathbb{R} \} \); range: \( \{y \in \mathbb{R}, y \leq -1\} \) |
| 2 | 3 |
| 3 | \( f(x) = 2|x - 3| + 1 \)  
   C |
| 4 |   |
| 5 | a vertical translation of \(-3\), a horizontal translation of \(-1\), and a vertical shrink by \( \frac{1}{2} \) |