Unit Overview

In this unit, students write the equations of quadratic functions to model situations and then graph these functions. They study methods of finding solutions to quadratic equations and interpreting these solutions. In the process, students learn about complex numbers.

Student Focus

Main Ideas for success in lessons 10-1, 10-2, & 10-3:

- Write equations of parabolas given a graph or key features
- Determine a quadratic function given three points on a plane
- Find a quadratic model given a set of data values
- Use a quadratic model to make predictions about data

Example:
Lesson 10-1:

The general equation for a parabola whose vertex is located at the origin, focus at (0, p), and directrix of \( y = -p \) is \( y = \frac{1}{4p} x^2 \).
Lesson 10-2:

What happens when you try to write the equation of the quadratic function that passes through the points (0, 4), (2, 2), and (4, 0)?

- You find that \( a = 0 \), \( b = -1 \), and \( c = 4 \), which results in the function \( f(x) = -x + 4 \). This function is linear, not quadratic.

What does this result indicate about the three points?

- The 3 points are on the same line, which means that you cannot write the equation of a quadratic function whose graph passes through the points.

Lesson 10-3:

**EXAMPLE**

Can the sequence 1, 8, 19, 34, 53, 76 be modeled by a quadratic function?

**Step 1:** Evaluate the first difference between the terms: 
\[
\begin{align*}
8 - 1 &= 7, \\
19 - 8 &= 11, \\
34 - 19 &= 15, \\
53 - 34 &= 19, \\
76 - 53 &= 23
\end{align*}
\]
Sequence of first differences: 7, 11, 15, 19, 23

**Step 2:** Evaluate the second difference by evaluating the difference between the first differences:
\[
\begin{align*}
11 - 7 &= 4, \\
19 - 11 &= 4, \\
23 - 19 &= 4
\end{align*}
\]
Sequence of second differences: 4, 4, 4

**Step 3:** Note that the sequence of second differences is constant. Therefore the sequence can be modeled by a quadratic function, specifically one having an \( x^2 \) coefficient of 2, since \( 2(2) = 4 \).

**Solution:** The sequence can be modeled by a quadratic function.

Explain how to determine the zeros of a quadratic function using a graphed quadratic model.

- Set the height \( y \) of the quadratic model equal to 0. Use the quadratic formula to solve for \( x \). The solutions will show when the parabola crosses the \( x \)-axis.
Unit Overview

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Student Focus

Main Ideas for success in lessons 11-1, 11-2, & 11-3:

→ Explore transformations of parabolas
→ Describes translations of quadratic parent functions
→ Write quadratic functions in vertex form

Example:

Lesson 11-1:

<table>
<thead>
<tr>
<th>Function</th>
<th>Transformation effect:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) = x^2 + 2 )</td>
<td>vertical stretch by a factor of 2</td>
</tr>
<tr>
<td>( h(x) = \frac{1}{2}x^2 )</td>
<td>vertical shrink by a factor of ( \frac{1}{2} )</td>
</tr>
<tr>
<td>( f(x) = -x^2 )</td>
<td>reflection over the x-axis</td>
</tr>
<tr>
<td>( k(x) = -\frac{1}{4}x^2 )</td>
<td>reflection over the x-axis AND vertical stretch by a factor of ( \frac{1}{4} )</td>
</tr>
<tr>
<td>( g(x) = (2x)^2 )</td>
<td>horizontal shrink by a factor of ( \frac{1}{2} )</td>
</tr>
<tr>
<td>( f(x) = \left(\frac{1}{4}x\right)^2 )</td>
<td>horizontal stretch by a factor of 4</td>
</tr>
</tbody>
</table>

Lesson 11-2:
Lesson 11-3:

Write \( f(x) = 3x^2 - 12x + 7 \) in vertex form.

**Step 1:** Factor the leading coefficient from the quadratic and linear terms.

\[
f(x) = 3(x^2 - 4x) + 7
\]

**Step 2:** Complete the square by taking half the linear coefficient \([0.5(-4) = -2]\), squaring it \([(−2)^2 = 4]\), and then adding it inside the parentheses.

\[
f(x) = 3\left(x^2 - 4x + \frac{4}{3}\right) + 7
\]

**Step 3:** To maintain the value of the expression, multiply the leading coefficient [3] by the number added inside the parentheses [4]. Then subtract that product [12].

\[
f(x) = 3(x^2 - 4x + 4) - 3(4) + 7
\]

\[
f(x) = 3(x^2 - 4x + 4) - 12 + 7
\]

**Step 4:** Write the trinomial inside the parentheses as a perfect square. The function is in vertex form.

\[
f(x) = 3(x - 2)^2 - 5
\]

**Solution:** The vertex form of \( f(x) = 3x^2 - 12x + 7 \) is \( f(x) = 3(x - 2)^2 - 5 \).
**Unit Overview**

In this unit, students write the equations of quadratic functions to model situations and then graph these functions. They study methods of finding solutions to quadratic equations and interpreting these solutions. In the process, students learn about complex numbers.

**Student Focus**

Main Ideas for success in lessons 12-1, 12-2, 12-3, 12-4, & 12-5:

- Graph quadratic equations and quadratic inequalities
- Write quadratic functions from verbal descriptions
- Identify and interpret key features of those functions
- Use the discriminant to determine that nature of the solutions of a quadratic equation.
- Use the discriminant to help graph a quadratic function.
- Graph quadratic inequalities
- Determine solutions to quadratic inequalities
Example:

Lesson 12-1:

Sandra sells tickets at the local skating center. She usually sells 500 tickets per day at $25 each when it is 30°F outside. She notices that for every increase by one degree in temperature, she sells 10 fewer tickets. Sandra reacts by increasing the ticket price by $0.50 for every degree over 30°F.

What function gives Sandra’s total sales, in dollars, as a function of the change in temperature, $x$?

- **A** $f(x) = (500 - x)(30 + x)$
- **B** $f(x) = (30 + x)(25 + 0.50x)$
- **C** $f(x) = (500 - 10x)(25 + 0.50x)$  
- **D** $f(x) = (25 + 0.50x)(500 - x)$

Lesson 12-2:

Math Tip: The reasonable domain and range of a function are the values in the domain and range of the function that make sense in a given real-world situation.

Example:
The Wilderness Scouts usually sell 400 boxes of granola bars at $4 each. But they discover that for every $0.25 increase in price, they lose 10 sales.

Which function gives their income as a function of $x$?

- **A** $f(x) = (4 + \frac{1}{4}x)(400 - x)$
- **B** $f(x) = (4 + \frac{1}{4}x)(400 - 10x)$  
- **C** $f(x) = (4 + x)(400 - x)$
- **D** $f(x) = (4 + x)(400 - 10x)$
Lesson 12-3:
Example: For the quadratic function \( f(x) = 2x^2 - 9x + 4 \), identify the vertex, the y-intercept, x-intercept(s), and the axis of symmetry. Graph the function.

<table>
<thead>
<tr>
<th>Identify ( a, b, ) and ( c ).</th>
<th>( a = 2, , b = -9, , c = 4 )</th>
</tr>
</thead>
</table>

**Vertex**

Use \(-\frac{b}{2a}\) to find the \( x \)-coordinate of the vertex.

\[
-\frac{(-9)}{2(2)} = \frac{9}{4} \quad \text{vertex: } \left(\frac{9}{4}, -\frac{49}{8}\right)
\]

Then use \( f\left(-\frac{b}{2a}\right)\) to find the \( y \)-coordinate.

**y-intercept**

Evaluate \( f(x) \) at \( x = 0 \).

\( f(0) = 4 \), so y-intercept is 4.

**x-intercepts**

Let \( f(x) = 0 \).

\[
2x^2 - 9x + 4 = 0
\]

Then solve for \( x \) by factoring or by using the Quadratic Formula.

\( x = \frac{1}{2} \) and \( x = 4 \) are solutions, so x-intercepts are \( \frac{1}{2} \) and 4.

**Axis of Symmetry**

Find the vertical line through the vertex,

\[
x = -\frac{b}{2a}
\]

\[
x = \frac{9}{4}
\]

**Graph**

Graph the points identified above: vertex, point on y-axis, points on x-axis.

Then draw the smooth curve of a parabola through the points.

The \( y \)-coordinate of the vertex represents the minimum value of the function. The minimum value is \(-498\).
Lesson 12-4:

<table>
<thead>
<tr>
<th>Discriminant of $ax^2 + bx + c = 0$</th>
<th>Solutions and x-intercepts</th>
<th>Sample Graph of $f(x) = ax^2 + bx + c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^2 - 4ac &gt; 0$</td>
<td>Two real solutions</td>
<td><img src="image1.png" alt="Graph 1" /></td>
</tr>
<tr>
<td>if $b^2 - 4ac$ is:</td>
<td>Two x-intercepts</td>
<td></td>
</tr>
<tr>
<td>a perfect square</td>
<td>roots are rational</td>
<td></td>
</tr>
<tr>
<td>not a perfect square</td>
<td>roots are irrational</td>
<td></td>
</tr>
</tbody>
</table>

| $b^2 - 4ac = 0$                      | One real rational solution (a double root) | ![Graph 2](image2.png) |
| One x-intercept                      |                                           |                         |

| $b^2 - 4ac < 0$                      | Two complex conjugate solutions         | ![Graph 3](image3.png) |
| No x-intercepts                      |                                           |                         |
Lesson 12-5:
Example:

Solve \( y > -x^2 - x + 6 \)

Graph the related quadratic function \( y = -x^2 - x + 6 \).
If the inequality symbol is > or <, use dotted curve.
If the symbol is \( \geq \) or \( \leq \), then use a solid curve.
This curve divides the plane into two regions.

Test \((0, 0)\) in \( y > -x^2 - x + 6 \).

\[ 0 > -0^2 - 0 + 6 \]

\[ 0 > 6 \] is a false statement.

Choose a point on the plane, but not on the curve, to test.

\((0, 0)\) is an easy point to use, if possible.

If the statement is true, shade the region that contains the point. If it is false, shade the other region.
The shaded region represents all solutions to the quadratic inequality.
Unit Overview

In this unit, students write the equations of quadratic functions to model situations and then graph these functions. They study methods of finding solutions to quadratic equations and interpreting these solutions. In the process, students learn about complex numbers.

Student Focus

Main Ideas for success in lessons 13-1 & 13-2:

- Solve systems of equations that include linear and nonlinear equations
- Look at solutions to systems of equations graphically
- Solve systems of equations algebraically

Example:

Lesson 13-1:
Graph the system of equations and identify the number of solutions.

\[ \begin{align*}
  y &= x \\
  y &= x^2 - 2
\end{align*} \]

Two real solutions.

\[ \begin{align*}
  y &= 2x - 3 \\
  y &= x^2 - 2
\end{align*} \]

One real solution.
Lesson 13-2:
The following system represents the supply and demand functions for basic haircuts at Salon Ultra Blue, where $y$ is the quantity of haircuts demanded or supplied when the price of haircuts is $x$ dollars. Solve this system algebraically to find the price at which the supply of haircuts equals the demand.

$$\begin{align*}
y &= -5x + 350 \\
y &= \frac{1}{10}x^2 - x - 5
\end{align*}$$

**Step 1:** Use substitution to solve for $x$.

\[
\begin{align*}
y &= -5x + 350 \\
-5x + 350 &= \frac{1}{10}x^2 - x - 5 \\
0 &= \frac{1}{10}x^2 + 4x - 355 \\
0 &= x^2 + 40x - 3550 \\
x &= \frac{-40 \pm \sqrt{40^2 - 4(1)(-3550)}}{2(1)} \\
x &= \frac{-40 \pm \sqrt{1600 + 14200}}{2} \\
x &= \frac{-40 \pm \sqrt{15800}}{2} \\
x &\approx -82.85 \text{ or } x \approx 42.85
\end{align*}
\]

**Step 2:** Substitute each value of $x$ into one of the original equations to find the corresponding value of $y$.

\[
\begin{align*}
y &= -5x + 350 \\
y &\approx -5(-82.85) + 350 \\
y &\approx 764 \\
y &= -5x + 350 \\
y &\approx -5(42.85) + 350 \\
y &\approx 136
\end{align*}
\]

**Step 3:** Write the solutions as ordered pairs.

The solutions are approximately $(-82.85, 764)$ and $(42.85, 136)$. Ignore the first solution because a negative value of $x$ does not make sense in this situation.

**Solution:** The price at which the supply of haircuts equals the demand is $42.85. At this price, customers will demand 136 haircuts, and the stylists will supply them.
**LESSON 7-1**

1. Consider a rectangle that has a perimeter of 80 cm.
   a. Write a function \( A(l) \) that represents the area of the rectangle with length \( l \).
   
   b. Graph the function \( A(l) \). Use an appropriate scale and label the axes.

   ![Graph](image)

   c. Is an area of 256 cm\(^2\) possible? How do you know? What is the length and width of the rectangle?

   d. What are the dimensions of a rectangle with a perimeter of 80 cm and an area of 300 cm\(^2\)?

   e. What are the reasonable domain and range of \( A(l) \)? Express your answers as inequalities, in interval notation, and in set notation.

   f. What is the greatest area that the rectangle can have? Explain. Give the dimensions of the rectangle with the greatest area and describe its shape.

2. A rectangle has a perimeter of 160 cm. What is the maximum area?
   A. 160 cm\(^2\)  
   B. 800 cm\(^2\)  
   C. 1600 cm\(^2\)  
   D. 6400 cm\(^2\)

3. Make sense of problems. Madison purchased 240 ft of fencing to build a corral for her horses. If each horse requires 600 ft\(^2\) of space, what is the maximum number of horses Madison can put in the corral she builds with the fencing? Explain.

4. Chance has 60 ft of fencing to build a dog pen. He plans to build the pen using one side of a 20-ft-long building. He will use all of the fencing for the other three sides of the pen. Use the area function for this rectangle to determine the area of the pen.

5. How is the maximum value of a quadratic function represented on the graph of the function?

**LESSON 7-2**

6. Factor \( x^2 - 12x + 27 \) by copying and completing the graphic organizer. Then check.

   ![Graphic Organizer](image)
7. Factor each quadratic expression.
   a. \(x^2 - 2x - 35\)  
   b. \(x^2 + 3x - 54\)
   c. \(x^2 - 4x + 4\)  
   d. \(x^2 - 121\)
   e. \(2x^2 + x - 15\)  
   f. \(6x^2 - 7x - 24\)
   g. \(3x^2 - 10x + 8\)  
   h. \(10x^2 + 9x - 9\)
   i. \(12x^2 + 25x + 12\)  
   j. \(15x^2 - x - 6\)

8. Which of the following is the factored form of 
   \(12x^2 - 15x - 18\)?
   A. \((6x - 3)(2x + 6)\)  
   B. \((2x - 3)(6x + 6)\)
   C. \((4x - 6)(3x + 3)\)  
   D. \((3x - 6)(4x + 3)\)

9. Reason abstractly. Given that \(b\) is positive and \(c\) is negative in the quadratic expression \(ax^2 + bx + c\), what can you conclude about the constant terms in the factored form?

10. Make use of structure. Can the quadratic expression \(x^2 + 2x + 2\) be factored using integers? Explain.

**LESSON 7-3**

11. Solve each equation by factoring.
   a. \(4x^2 + 4x - 3 = 0\)  
   b. \(3x^2 - x - 10 = 0\)
   c. \(3x^2 - 8x + 4 = 0\)  
   d. \(4x^2 - 8x - 5 = 0\)
   e. \(6x^2 + 5x - 6 = 0\)  
   f. \(5x^2 + 2x = 3\)
   g. \(8x^2 + 3 = -10x\)  
   h. \(6x^2 + 5 = 11x\)
   i. \(7x^2 + 15x = 18\)  
   j. \(10x^2 - 12 = 7x\)

12. For each set of solutions, write a quadratic equation in standard form.
   a. \(x = 3, x = -2\)  
   b. \(x = -4, x = -1\)
   c. \(x = 5, x = -3\)  
   d. \(x = 7, x = -3\)
   e. \(x = -\frac{1}{2}, x = 1\)  
   f. \(x = \frac{2}{3}, x = \frac{1}{2}\)
   g. \(x = \frac{3}{4}, x = -\frac{2}{3}\)  
   h. \(x = \frac{3}{5}, x = \frac{1}{3}\)

13. Julio has 200 ft of fencing to put around a field that has an area of 2500 sq ft. Which equation can be used to find the length of the field?
   A. \(l^2 - 100l + 2500 = 0\)  
   B. \(l^2 - 200l + 2500 = 0\)
   C. \(l^2 - 100l - 2500 = 0\)  
   D. \(l^2 - 50l + 2500 = 0\)

14. Make use of structure. What property do you use to solve a quadratic equation by factoring? Explain.

15. Make sense of problems. Akiko wants to fence in her 12 ft by 10 ft vegetable garden. There will be a path \(x\) ft wide between the garden and the fence. The area to be enclosed by the fence will be 360 sq ft.
   a. Model with mathematics. Draw a diagram of the situation.
   b. Write a quadratic equation that can be used to determine the value of \(x\).
   c. Solve the equation by factoring.
   d. Interpret the solutions.
LESSON 7-4

16. For what values of $x$ is the product $(x + 5)(x - 1)$ positive? Explain.

17. Use the number line provided to solve each inequality.
   a. $2x^2 - 5x - 12 \geq 0$
   b. $x^2 + 2x - 8 < 0$

20. Model with mathematics. Simon wants to enclose a rectangular corral next to the barn. The side of the barn will form one side of the corral. The other three sides will be fencing. Simon purchased 150 ft of fencing and wishes to enclose an area of at least 2500 sq ft.
   a. Write an inequality in terms of $l$ that represents the possible area of the pen.
   b. Write the inequality in standard form with integer coefficients.
   c. Factor the inequality.
   d. Determine the possible lengths and widths of the corral.

LESSON 8-1

21. Write each number in terms of $i$.
   a. $\sqrt{-36}$
   b. $\sqrt{-121}$
   c. $\sqrt{-5}$
   d. $\sqrt{-24}$
   e. $\sqrt{-27}$
   f. $\sqrt{-98}$
   g. $\sqrt{-48}$
   h. $\sqrt{-900}$

22. Make use of structure. Which of the following numbers can be written as $5i$?
   A. $\sqrt{-25}$
   B. $-\sqrt{25}$
   C. $\sqrt{25i}$
   D. $\sqrt{-25i}$
23. Graph each complex number on the complex plane.
   a. $4 + 3i$
   b. $3 - i$
   c. $-4i$
   d. $-2 - 2i$

24. Name the complex number represented by each labeled point on the complex plane.

25. The sum of two numbers is 12, and their product is 40.
   a. Let $x$ represent one of the numbers, and write an expression for the other number in terms of $x$. Use the expression to write an equation that models the situation given above.
   b. Use the Quadratic Formula to solve the equation. Write the solutions in terms of $i$.

LESSON 8-2

26. Find each sum or difference.
   a. $(3 + 7i) + (9 - 5i)$
   b. $(5 - 3i) - (6 - 7i)$
   c. $(-2 + 8i) + (7 - 3i)$
   d. $(15 + 6i) - (-9 - 4i)$
   e. $\left(\frac{1}{2} + 3i\right) - \left(\frac{3}{2} + 5i\right)$
   f. $(3\sqrt{2} - 5i) - (2\sqrt{2} + 6i)$
   g. $(3\sqrt{5} + 4i) - (3\sqrt{5} - 4i)$
   h. $2i + (3 - 2i)$

27. Multiply. Write each product in the form $a + bi$.
   a. $(2 + 5i)(3 - 2i)$
   b. $(8 - 3i)(2 + i)$
   c. $(5 + 2i)(5 - 2i)$
   d. $(7 + 3i)(-3 + 5i)$
   e. $(3 - 4i)(6 - 2i)$
   f. $(2 + 5i)(3 - 4i)$
   g. $(-1 - 5i)(3 + 2i)$
   h. $(5 + 4i)(2 - 3i)$
28. Divide. Write each quotient in the form $a + bi$.
   a. $\frac{3 - 2i}{5 + i}$
   b. $\frac{2 + 3i}{4 - 2i}$
   c. $\frac{3 + i}{3 - i}$
   d. $\frac{8 - 7i}{1 - 2i}$
   e. $\frac{1 + 4i}{1 - i}$
   f. $\frac{3 - 5i}{2i}$
   g. $\frac{5}{2 + 3i}$
   h. $\frac{5 + i}{3 - i}$

29. **Make use of structure.** Give an example of a complex number you could subtract from $5 - 2i$ that would result in a real number. Show that the difference of the complex numbers is equal to a real number.

30. Which of the following is the complex conjugate of $-3 + 7i$?
   A. $3 + 7i$
   B. $3 - 7i$
   C. $-3 - 7i$
   D. $-3 + 7i$

**LESSON 8-3**

31. Use complex conjugates to factor each expression.
   a. $9x^2 + 36$
   b. $25x^2 + 49$
   c. $3x^2 + 27y^2$
   d. $5x^2 + 100y^2$

32. Solve each equation by factoring.
   a. $x^2 + 25 = 0$
   b. $16x^2 + 4 = 0$
   c. $4x^2 + 20 = 0$
   d. $8x^2 = -36$

33. Which are the solutions of the quadratic function $f(x) = 4x^2 + 9$?
   A. $x = -\frac{1}{3}, x = \frac{1}{3}$
   B. $x = -\frac{4}{9}, x = \frac{4}{9}$
   C. $x = -\frac{2}{3}, x = \frac{2}{3}$
   D. $x = -\frac{3}{2}, x = \frac{3}{2}$

34. What are the solutions of each quadratic function?
   a. $x^2 + 49$
   b. $25x^2 + 16$
   c. $9x^2 + 64$
   d. $100x^2 + 81$

35. **Attend to precision.** What are the solutions of the equation $9x^2 + 32$? Show your work.

**LESSON 9-1**

36. **Attend to precision.** Solve the equation $3(x - 2)^2 - 7 = 0$, and explain each of your steps.

37. Solve for $x$.
   a. $16x^2 - 25 = 0$
   b. $9x^2 - 5 = 0$
   c. $8x^2 - 9 = 0$
   d. $3x^2 + 49 = 0$
   e. $4(x - 3)^2 - 81 = 0$
   f. $5(x + 9)^2 + 16 = 0$
   g. $3(x - 4)^2 - 10 = 0$
   h. $7(x + 2)^2 + 9 = 0$

38. Which is NOT a perfect square trinomial?
   A. $x^2 + 10x + 25$
   B. $x^2 - 12x + 36$
   C. $x^2 + 16x + 81$
   D. $4x^2 + 8x + 4$
39. Use the method of completing the square to make a perfect square trinomial. Then factor the perfect square trinomial.
   a. \( x^2 + 8x \)  
   b. \( x^2 - 14x \)

40. Solve for \( x \) by completing the square.
   a. \(-8x = x^2 - 1\)  
   b. \(3x^2 - 18x + 2 = 0\)
   c. \(x^2 - 10x + 3 = 0\)  
   d. \(2x^2 - 12x + 21 = 0\)

**LESSON 9-2**

41. Write the Quadratic Formula.

42. Solve each equation using the Quadratic Formula.
   a. \(2x^2 + x - 1 = 0\)  
   b. \(x^2 = 2x + 2\)
   c. \(4x^2 + 4x = 7\)  
   d. \(2x^2 - 5x + 2 = 0\)
   e. \(3x^2 + 7x - 6 = 0\)  
   f. \(10x^2 - 5x + 1 = 0\)
   g. \(3x^2 + 11x - 17 = 0\)  
   h. \(x^2 + 13x - 9 = 0\)

43. Which quadratic equation would be best solved by using the Quadratic Formula to find the solutions?
   A. \(4x^2 - 81 = 0\)  
   B. \(x^2 + x - 6 = 0\)
   C. \(x^2 + 36 = 0\)  
   D. \(3x^2 + 4x + 5 = 0\)

44. Solve each quadratic equation by using any of the methods you have learned. For each equation, tell which method you used and why you chose that method.
   a. \(x^2 + 6x = 9\)  
   b. \(x^2 - 2x - 15 = 0\)
   c. \(5x^2 + 9x - 4 = 0\)  
   d. \((x - 5)^2 - 81 = 0\)

45. Gaetano shoots a basketball from a height of 6.5 ft with an initial vertical velocity of 17 ft/s. The equation \(-16t^2 + 17t + 6.5 = 10\) can be used to determine the time \(t\) in seconds at which the ball will have a height of 10 ft, the same height as the basket.
   a. Solve the equation by using the Quadratic Formula.
   b. **Attend to precision.** To the nearest tenth of a second, when will the ball have a height of 10 ft?
   c. Explain how you can check that your answers to part b are reasonable.
LESSON 9-3

46. Write the discriminant of the quadratic equation \( ax^2 + bx + c = 0 \). Explain how it is used.

47. For each equation, compute the value of the discriminant and describe the solutions without solving the equation.
   a. \( x^2 + 3x - 7 = 0 \) 
   b. \( 2x^2 - 5x + 9 = 0 \)
   c. \( 5x^2 + 6x + 1 = 0 \) 
   d. \( x^2 - 6x = -9 \)

48. The discriminant of a quadratic equation is less than 0. What is the nature of the solutions of the equation?
   A. one rational solution 
   B. two complex conjugate solutions 
   C. two irrational solutions 
   D. two rational solutions

49. A quadratic equation has one real, rational solution.
   a. What is the value of the discriminant?
   b. Reason abstractly. Give an example of a quadratic equation that has one real, rational solution.

50. a. Under what circumstances will the radicand in the Quadratic Formula be positive?
   b. What does this tell us about the solutions?
   c. When will the solutions be rational?

LESSON 10-1

51. Which equation does the graph represent?

A. \( y = 2(x - 1)^2 + 3 \) 
B. \( y = 2(x - 1)^2 - 3 \)
C. \( y = 2(x + 1)^2 + 3 \) 
D. \( y = 2(x + 1)^2 - 3 \)

52. A parabola has a focus of \((-2, 3)\) and a directrix of \( y = 1 \). Answer each question about the parabola, and explain your reasoning.
   a. What is the axis of symmetry?
   b. What is the vertex?
   c. In which direction does the parabola open?
53. **Reason quantitatively.** Use the given information to write the equation of each parabola.

   a. vertex: \((5, -3)\); directrix: \(x = 2\)

   b. focus: \((-2, 4)\); directrix: \(y = 2\)

   c. axis of symmetry: \(x = 0\); vertex: \((0, 0)\);
   directrix: \(y = 4\)

   d. focus: \((-3, 5)\); vertex: \((-1, 5)\)

54. The equation of a parabola is \(y = \frac{1}{4}(x + 3)^2 - 1\). Identify the vertex, axis of symmetry, focus, and directrix of the parabola.

55. Graph the parabola given by the equation
\[ y = \frac{1}{2}(x + 3)^2 - 1. \]

56. Write the equation of the quadratic function whose graph passes through each set of points.

   a. \((-1, 12), (1, 4), (2, 3)\)

   b. \((0, -1), (2, 7), (3, 14)\)

   c. \((-2, 11), (1, 2), (3, 6)\)

   d. \((-2, -3), (0, 1), (1, 6)\)

57. The table below shows the first few terms of a quadratic function. Write a quadratic equation in standard form that describes the function.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>(-7)</td>
<td>(-5)</td>
<td>(-1)</td>
<td>(5)</td>
<td>(13)</td>
</tr>
</tbody>
</table>

58. Which equation describes the parabola that passes through the three points \((0, 14), (3, -4)\), and \((5, 4)\)?

   A. \(y = x^2 - 12x + 14\)
   
   B. \(y = 2x^2 - 12x + 14\)
   
   C. \(y = 2x^2 - 6x + 7\)
   
   D. \(y = x^2 - 6x - 4\)
59. Graph the quadratic function that passes through the points \((0, 5), (-2, 11), \text{ and } (2, 3)\).

60. **Reason quantitatively.** The graph of a quadratic function passes through the point \((2, -2)\). The vertex of the graph is \((1, -3)\).
   a. Use symmetry to identify another point on the graph of the function. Explain how you determined your answer.
   b. Write the equation of the quadratic function in standard form.

**LESSON 10-3**

61. Tell whether a linear model or a quadratic model is a better fit for each data set. Justify your answer.
   a. 
   \[
   \begin{array}{c|cccccccc}
   x & 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 \\
   y & 10 & 12 & 15 & 18 & 20 & 22 & 20 & 19 \\
   \end{array}
   \]

   b. 
   \[
   \begin{array}{c|cccccccc}
   x & 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 \\
   y & 10 & 21 & 33 & 44 & 56 & 67 & 78 & 90 \\
   \end{array}
   \]

62. The tables show time and height data for two rockets.

   **Rocket A**
   \[
   \begin{array}{c|cccccccc}
   \text{Time (s)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
   \text{Height (m)} & 0 & 41 & 98 & 137 & 191 & 215 & 238 & 279 \\
   \end{array}
   \]

   **Rocket B**
   \[
   \begin{array}{c|cccccccc}
   \text{Time (s)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
   \text{Height (m)} & 0 & 57 & 183 & 248 & 290 & 325 & 362 & 385 \\
   \end{array}
   \]

   a. **Use appropriate tools strategically.** Use a graphing calculator to perform a quadratic regression for each data set. Write the equations of the quadratic models. Round the coefficients and constants to the nearest tenth.
   b. Use your models to predict which rocket had a greater maximum height. Explain.
   c. Use your models to predict which rocket hit the ground first and how much sooner.

63. Which quadratic model is the best fit for the data in the table? Use your calculator.

   \[
   \begin{array}{c|cccccccc}
   x & 20 & 24 & 28 & 32 & 36 & 40 & 44 & 48 \\
   y & 12 & 19 & 21 & 37 & 23 & 18 & 15 & 11 \\
   \end{array}
   \]

   A. \(y = -0.1x^2 + 5.5x - 64\)
   B. \(y = 0.1x^2 + 5.5x - 64\)
   C. \(y = 0.2x^2 - 1.2x + 48.3\)
   D. \(y = 6.4x^2 + 5.5x - 0.1\)

64. What is the least number of points that are needed to perform a quadratic regression on a graphing calculator? Explain.
65. The Quality Shoe Company tests different prices of a new type of shoe at different stores. The table shows the relationship between the selling price of a pair of shoes and the monthly revenue per store the company made from selling the shoes.

<table>
<thead>
<tr>
<th>Selling Price ($)</th>
<th>Monthly Revenue per Store ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>8500</td>
</tr>
<tr>
<td>150</td>
<td>9300</td>
</tr>
<tr>
<td>200</td>
<td>10,400</td>
</tr>
<tr>
<td>250</td>
<td>11,650</td>
</tr>
<tr>
<td>300</td>
<td>10,500</td>
</tr>
<tr>
<td>350</td>
<td>9800</td>
</tr>
<tr>
<td>400</td>
<td>8900</td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to determine the equation of a quadratic model that can be used to predict $y$, the monthly revenue per store in dollars when the selling price for each pair of shoes is $x$ dollars. Round values to the nearest hundredth.

b. Is a quadratic model a good model for the data set? Explain.

c. Use your model to determine the price at which the company should sell the shoes to generate the greatest revenue.

---

66. Describe each function as a transformation of $f(x) = x^2$. Then use the information to graph each function on the coordinate grid.

a. $f(x) = x^2 - 3$

b. $f(x) = (x - 3)^2$
67. Each function graphed below is a transformation of $f(x) = x^2$. Describe each transformation and write the equation of the transformed function.

a.

b.

c.

d. $f(x) = (x - 2)^2 - 3$

c. $f(x) = (x + 2)^2 + 3$
68. Make use of structure. \( p(x) = (x + 1)^2 - 2 \) is a transformation of \( f(x) = x^2 \). Which is a description of the transformation?

A. translation 1 unit to the right and 2 units down  
B. translation 1 unit to the left and 2 units down  
C. translation 2 units to the right and 1 unit up  
D. translation 2 units to the left and 1 unit up

69. What is the vertex of the function \( g(x) = (x + 2)^2 - 4 \)? Justify your answer in terms of a translation of \( f(x) = x^2 \).

70. What is the axis of symmetry of the function \( h(x) = (x - 3)^2 + 1 \)? Justify your answer in terms of a translation of \( f(x) = x^2 \).

Lesson 11-2

71. Describe the graph of each function as a transformation of the graph of \( f(x) = x^2 \). Then use the information to graph each function on the coordinate plane.

a. \( f(x) = -x^2 + 3 \)

b. \( f(x) = 2x^2 - 1 \)
c. \( f(x) = \left( \frac{1}{2}x \right)^2 - 3 \)

72. Each function graphed below is a transformation of \( f(x) = x^2 \). Describe the transformation and write the equation of the transformed function below each graph.

a.

b.
73. Make use of structure. Describe how the graph of \( g(x) = \frac{1}{2}x^2 \) differs from the graph of \( h(x) = \left( \frac{1}{2}x \right)^2 \).

74. \( f(x) = x^2 \) is translated 1 unit down, shrunk by a factor of \( \frac{1}{2} \), and reflected over the \( x \)-axis. Which is the equation of the transformation?
   - A. \( g(x) = \left( \frac{1}{2}x - 1 \right)^2 \)
   - B. \( g(x) = -\left( \frac{1}{2}x - 1 \right)^2 \)
   - C. \( g(x) = -\frac{1}{2}x^2 - 1 \)
   - D. \( g(x) = \frac{1}{2}x^2 - 1 \)

75. Without graphing, determine the vertex of the graph of \( h(x) = 3(x + 1)^2 + 2 \). Explain how you found your answer.

**LESSON 11-3**

76. Write each quadratic function in vertex form. Then describe the transformation(s) from the parent function and use the description to graph the function.
   - a. \( g(x) = x^2 - 6x + 14 \)
b. \( h(x) = x^2 + 10x + 24 \)

\[
\begin{array}{c}
h(x) \\
\end{array}
\]

77. What is the vertex of the graph of the function
\( f(x) = 2(x + 3)^2 + 5 \)? Explain your answer.

78. Write each function in vertex form. Then identify the vertex and axis of symmetry of the function's graph, and tell in which direction the graph opens.

a. \( h(x) = x^2 + 4x \)

vertex form: 
vertex: 
axis of symmetry: 
graph opens:

b. \( h(x) = x^2 - 8x + 19 \)

vertex form: 
vertex: 
axis of symmetry: 
graph opens:
c. \( h(x) = -2x^2 + 12x - 17 \)
   vertex form: __________________
   vertex: __________
   axis of symmetry: __________
   graph opens: _______________

d. \( h(x) = 3x^2 + 24x + 40 \)
   vertex form: __________________
   vertex: __________
   axis of symmetry: __________
   graph opens: _______________

79. Which function has an axis of symmetry to the left of the \( y \)-axis?
   A. \( f(x) = x^2 + 8x + 10 \)
   B. \( f(x) = x^2 - 6x + 10 \)
   C. \( f(x) = x^2 - 2x + 3 \)
   D. \( f(x) = -x^2 + 4x + 8 \)

80. Express regularity in repeated reasoning. Sal is writing \( f(x) = 3x^2 - 6x - 5 \) in vertex form. What number should he write in the first box below to complete the square inside the parentheses? What number should he write in the second box to keep the expression on the right side of the equation balanced? Explain.
   \[ f(x) = 3(x^2 - 2x + \square) - \square - 5 \]

82. Construct viable arguments. Suppose you are asked to find the vertex of the graph of \( f(x) = 2x^2 - 12x + 27 \). Explain your answer. Find the vertex and explain how you found the vertex using your chosen method.

83. Use the formula for the \( x \)-coordinate of the vertex to find the vertex of each function.
   a. \( f(x) = x^2 + 10x + 15 \)
   b. \( f(x) = 2x^2 - 28x + 81 \)
   c. \( f(x) = 3x^2 - 18x + 28 \)
   d. \( f(x) = 2x^2 + 4x + 5 \)

LESSON 12-1

81. The graph of a quadratic function \( f(x) \) opens downward, and its vertex is \((3, -2)\). For what values of \( x \) does the value of \( f(x) \) increase? For what values of \( x \) does the value of \( f(x) \) decrease? Explain your answers.
84. Which is the vertex of \( f(x) = 2x^2 + 20x + 30? \)
   A. (20, 30)  
   B. (−5, −20)  
   C. (20, 5)  
   D. (5, −20)

85. Mrs. Miller would like to create a small vegetable garden adjacent to her house. She has 20 ft of fencing to put around three sides of the garden.
   a. Let \( x \) be the width of the garden. Write the standard form of a quadratic function \( G(x) \) that gives the area of the garden in square feet in terms of \( x \).
   b. Graph \( G(x) \) and label the axes.

\[
\begin{array}{c}
\text{G(x)} \\
\hline
\hline
\end{array}
\]

\( x \)-intercepts:  
\( y \)-intercept:  

b. What is the vertex of the graph of \( G(x) \)? What do the coordinates of the vertex represent in this situation?

d. **Reason quantitatively.** What are the dimensions of the garden that yield the maximum area? Explain your answer.
87. What is the x-intercept of a quadratic function? How many x-intercepts can a quadratic function have?

88. Which of the following are the x-intercepts of \( f(x) = 6x^2 - 4x - 10 \)?

A. -5 and 2
B. -10 and 0
C. -1 and \( \frac{5}{3} \)
D. 2 and 3

89. When does the graph of a quadratic function have only one x-intercept?

90. You can buy a 24-hour ticket for the Hop-On Hop-Off bus tour in London for £20. (The basic unit of money in the United Kingdom is the pound, £.) The tour company is considering increasing the cost of the ticket to increase the profit. If the tickets are too expensive, they will not have any customers. The function \( T(x) = -x^2 + 50x - 225 \) models the profit the tour company makes by selling tickets for \( x \) pounds each.

a. What is the y-intercept of the graph of \( T(x) \), and what is its significance?

b. What are the x-intercepts of the graph of \( T(x) \), and what is their significance?

c. Give the reasonable domain and range of \( T(x) \), assuming that the tour company does not want to lose money by selling the tickets. Explain how you determined the reasonable domain and range.

d. Make sense of problems. What selling price for the tickets would maximize the tour company’s profit? Explain your answer.

**LESSON 12-3**

91. For each function, identify the vertex, y-intercept, x-intercept(s), and axis of symmetry. Graph the function. Identify whether the function has a maximum or minimum and give its value.

a. \( f(x) = 2x^2 - 7x + 5 \)

vertex: _______________

y-intercept: _______________

x-intercept(s): _______________

axis of symmetry: _______________

max or min: _______________

b. Graph of \( f(x) \)
b. \( f(x) = -x^2 + 2x + 3 \)

vertex: ________________

\( y \)-intercept: ________________

\( x \)-intercept(s): ________________

axis of symmetry: ________________

max or min: ________________

92. **Make sense of problems.** Consider the London bus tour company function \( T(x) = -x^2 + 50x - 225 \) whose graph is below.

a. Based on the model, what selling price(s) would result in a profit of £300? Explain how you determined your answer.

b. Could the tour company make £500? Explain.

c. If the tour company sells the tickets for £10 each, how much profit can it expect to make? Explain how you determined your answer.

93. Explain how to find the \( y \)-intercept of the quadratic function \( f(x) = x^2 - 60x + 35 \) without graphing the function.

94. If a parabola opens down, then the \( y \)-coordinate of the vertex is the

A. minimum value  
B. axis of symmetry  
C. \( y \)-intercept  
D. maximum value

95. Suppose you are given the vertex of a parabola. How can you find the axis of symmetry?
LESSON 12-4

96. What does the discriminant of a quadratic function tell you about the \( x \)-intercepts of the graph of the function?

97. The discriminant of a quadratic equation is greater than zero but not a perfect square. What is the nature of the solutions? How many \( x \)-intercepts will the graph of the equation have?

98. For each equation, find the value of the discriminant and describe the nature of the solutions. Then graph the related function and find the \( x \)-intercepts if they exist.

a. \( x^2 + 3x - 5 = 0 \)

value of the discriminant: \\
nature of the solutions: \\
x-intercepts: 

b. \( x^2 - 4x + 4 = 0 \)

value of the discriminant: \\
nature of the solutions: \\
x-intercepts: 

c. \( 2x^2 + 3x + 5 = 0 \)

value of the discriminant: \\
nature of the solutions: \\
x-intercepts: 

d. $3x^2 + 10x + 3 = 0$

value of the discriminant: __________
nature of the solutions: __________
x-intercepts: __________________________

99. If the discriminant is zero, what is true about the solution(s) of the quadratic equation?
   A. There are two complex conjugate solutions.
   B. There are two real solutions.
   C. There is one, rational solution.
   D. There are no solutions.

100. Construct viable arguments. A quadratic equation has two complex conjugate solutions. What can you conclude about the value of the discriminant of the equation?

101. Solve each quadratic inequality by graphing.
   a. $y \geq -\frac{1}{2}x^2 + 4x - 5$
   b. $y > x^2 - 4$
c. \( y \leq \frac{1}{2}x^2 + x - 3 \)

\[ y \]
\[ x \]

\[ y \]
\[ x \]

\[ y \]
\[ x \]

d. \( y < -x^2 - 4x + 1 \)

\[ y \]
\[ x \]

\[ y \]
\[ x \]

102. Which inequality’s solutions are shown in the graph?
A. \( y \leq -2x^2 + 8x - 7 \)
B. \( y < -2x^2 + 8x - 7 \)
C. \( y \geq -2x^2 + 8x - 7 \)
D. \( y > -2x^2 + 8x - 7 \)

103. Which of the following is NOT a solution of the inequality \( y \geq x^2 - 3x + 5 \)?
A. \( (0, 5) \)
B. \( (3, 3) \)
C. \( (5, 15) \)
D. \( (-1, 9) \)
104. Graph the quadratic inequality \( y < -x^2 - 6x - 13 \). Then state whether each ordered pair is a solution of the inequality.

a. \((0, -2)\)  
b. \((-3, -4)\)  
c. \((-3, -6)\)  
d. \((-4, -12)\)

105. **Model with mathematics.** Foresters use a measure known as diameter at breast height (DBH) to measure trees for logging. To find the DBH, they use diameter tape or a caliper to measure the tree at a height of 4.5 feet (breast height) from the ground to determine the diameter. A tree of a certain species should have a cross-sectional area of at least 10 square feet at breast height for it to be logged. Suppose that mature trees for this species do not have a cross-sectional area of more than 30 square feet at this height.

a. Write the function for the DBH in terms of the radius, \( r \). Is this a linear function or a quadratic function? Explain.

b. Write the function for the area which is acceptable for logging in terms of the radius, \( r \). Is this a linear function or a quadratic function? What is the reasonable domain and range of the function?

106. Graph each system. Write the solution(s) if any.

a. \[ \begin{align*} 
    y &= x + 7 \\
    y &= x^2 - 1 
\end{align*} \]

b. \[ \begin{align*} 
    y &= -x - 3 \\
    y &= -x^2 + 3 
\end{align*} \]
107. **Critique the reasoning of others.** Mari claims that a system of a linear equation and a quadratic equation can have one solution. Zelly says that the system has to have two solutions. Who is correct? Explain using a system and a graph as an example.

108. The demand function for a product is \( f(x) = -3x + 25 \). The supply function is \( g(x) = \frac{1}{5}x^2 - 2x + 3 \). Use a graphing calculator to determine the solution(s) to the system.

A. \((64.9, -13.3)\) and \((0.2, 8.3)\)
B. \((-13.3, 64.9)\) and \((8.3, 0.2)\)
C. \((13.3, -64.9)\) and \((-8.3, -0.2)\)
D. \((-64.9, 13.3)\) and \((-0.2, -8.3)\)
109. Aaron sells T-shirts at the Jazz Festival in New Orleans. He decides to lower the price of the T-shirts.
   a. How might this affect the demand for the T-shirts?

   b. Will he realize an increase in profit? Explain.

   c. What will be the break-even point, the point where revenue from sales covers the cost? Explain.

110. Use appropriate tools strategically. Cathie wrote the following system of equations to model a problem in her research project.

\[
\begin{align*}
    f(x) &= -10x + 700 \\
    g(x) &= 0.3x^2 - 24.4x + 569
\end{align*}
\]

Sketch a graph of the system and identify the solution(s).

111. Express regularity in repeated reasoning. Write a system of equations that consists of one linear equation and one quadratic equation. Explain how you would solve the system algebraically.

112. Find the real solutions of each system algebraically.

   a. \(
      \begin{align*}
         y &= 3x - 7 \\
         y &= x^2 + 6x - 17
      \end{align*}
   \)

   b. \(
      \begin{align*}
         y &= 2x + 7 \\
         y &= x^2 - 6x + 25
      \end{align*}
   \)

   c. \(
      \begin{align*}
         y &= 4x - 11 \\
         y &= x^2 - 3x - 1
      \end{align*}
   \)

   d. \(
      \begin{align*}
         y &= 2x - 5 \\
         y &= x^2 - 2x + 2
      \end{align*}
   \)

113. How many real solutions does the following system have?

\[
\begin{align*}
    y &= 3x - 7 \\
    y &= x^2 + 11x + 9
\end{align*}
\]

A. none  \quad B. one  \quad C. two  \quad D. infinitely many

114. Describe the solutions of the system of equations from Item 112b.

115. Confirm the solutions to Item 112c by graphing. Describe the solution(s).