Unit Overview

In this unit, students study arithmetic and geometric sequences and implicit and explicit rules for defining them. Then they analyze exponential and logarithmic patterns and graphs as well as properties of logarithms. Finally, they solve exponential and logarithmic equations.

Student Focus

Main Ideas for success in lessons 21-1, 21-2, 21-3, 21-4, & 21-5:

→ Examine exponential functions and their graphs
→ Investigate linear growth and decay
→ Compare rates of change in exponential and linear data
→ Write exponential functions
→ Transform parent exponential functions
→ Examine base exponential functions

Example:

Lesson 21-1:

Math Tips:
Linear functions have the property that the rate of change of the output variable, \( y \) with respect to the input variable, \( x \) is constant, that is, the ratio \( \frac{\Delta y}{\Delta x} \) is constant for linear functions.

Exponential functions have the property that the rate of change is a constant multiplier such that \( f(x) = ab^x \) where \( a \) and \( b \) are constants, \( x \) is the domain, \( f(x) \) is the range, and \( a \neq 0, b > 0, b \neq 1 \).

Example 1:
What values of \( a \) and \( b \) in this table represents a linear function?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>( a )</td>
</tr>
<tr>
<td>4</td>
<td>( b )</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
</tbody>
</table>

Answer: \( \text{rate of change} = \frac{6}{1} \) \( a = 28 \), \( b = 34 \)

Example 2:
What value of \( k \) in this table represents an exponential function?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>576</td>
</tr>
<tr>
<td>53</td>
<td>432</td>
</tr>
<tr>
<td>54</td>
<td>( k )</td>
</tr>
<tr>
<td>55</td>
<td>243</td>
</tr>
</tbody>
</table>

Answer: \( \frac{y_1}{y_2} = \frac{4}{3} \) \( k = 324 \)
Lesson 21-2:
In an exponential function, the constant multiplier, or scale factor, is called an exponential decay factor when the constant is less than 1. When the constant is greater than 1, it is called an exponential growth factor.

Example 1:
What type of function is represented by the table and for what reason?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>640</td>
</tr>
<tr>
<td>6</td>
<td>10,240</td>
</tr>
</tbody>
</table>

Answer: exponential because the y-values for successive integer values of x have a common ratio of 4.

Example 2:
Brittany bought a new car for $20,000. Cars lose value over time. After 1 year, it was worth $18,000. After two years, it was worth $16,200. What is the decay value for her car?

Answer: decay 0.9

Lesson 21-3:
<table>
<thead>
<tr>
<th>Exponential Function</th>
<th>Increase/ Decrease?</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 2^x$</td>
<td>increasing</td>
<td>y-intercept different from $g(x)$ and $j(x)$</td>
</tr>
<tr>
<td>$g(x) = 3(2)^x$</td>
<td>increasing</td>
<td>y-intercept different from $f(x)$ and $j(x)$, reflection over the x-axis of $h(x)$.</td>
</tr>
<tr>
<td>$h(x) = -3(2)^x$</td>
<td>decreasing</td>
<td>reflection over the x-axis of $g(x)$</td>
</tr>
<tr>
<td>$j(x) = \frac{1}{4}(2)^x$</td>
<td>increasing</td>
<td>y-intercept different from $g(x)$ and $h(x)$, reflection over the x-axis of $k(x)$.</td>
</tr>
<tr>
<td>$k(x) = -\frac{1}{4}(2)^x$</td>
<td>decreasing</td>
<td>reflection over the x-axis of $j(x)$.</td>
</tr>
</tbody>
</table>

Increasing exponential functions have a horizontal asymptote as $x$ approaches $-\infty$, decreasing exponential functions have a horizontal asymptote as $x$ approaches $+\infty$.

Lesson 21-4:
Compare the parent function to the transformed exponential function.

<table>
<thead>
<tr>
<th>Parent function: $f(x) = 10^x$</th>
<th>$g(x) = 2(10)^x$</th>
<th>vertical stretch by a factor of 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x) = 10^{(x-1)}$</td>
<td>horizontal translation of the parent function to the right 1 unit</td>
<td></td>
</tr>
<tr>
<td>$j(x) = 10^x + 3$</td>
<td>vertical translation of the parent function up 3 units.</td>
<td></td>
</tr>
<tr>
<td>$k(x) = \frac{1}{10}$</td>
<td>$k(x)$ has horizontal asymptote as $x$ approaches $+\infty$</td>
<td></td>
</tr>
</tbody>
</table>

Lesson 21-5:
Which describes the function $f(x) = e^x$?

a) x-intercept none, y-intercept (1, 0)
b) x-intercept (1, 0), y intercept (0, 1)
c) x-intercept none, y-intercept (0, 1)
d) x-intercept (1, 0), y-intercept none
Unit Overview

In this unit, students study arithmetic and geometric sequences and implicit and explicit rules for defining them. Then they analyze exponential and logarithmic patterns and graphs as well as properties of logarithms. Finally, they solve exponential and logarithmic equations.

Student Focus

Main Ideas for success in lessons 22-1, 22-2, 22-3, & 22-4

→ Examine logarithmic functions and their graphs
→ Review exponential functions
→ Examine the relationship between logarithmic and exponential functions
→ Write equations using logarithmic and exponential forms

Example:

Lesson 22-1:

Example 1:
Sarah wants to graph the exponential function that includes the two points in the table. She wants to show the value of the function when x = 4. Sarah marks the y-axis in multiples of 100. What is the smallest maximum y-value that will show all three points on Sarah’s graph?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>136</td>
</tr>
</tbody>
</table>

a) 500  
b) 600  
c) 700  
d) 800

Example 2:
What are the domain and range of $f(x) = \frac{1}{2}(2)^x$?

a) domain $(0, \infty)$, range $[0, \infty)$

b) domain $(-\infty, \infty)$, range $[0, \infty)$

c) domain $(0, \infty)$, range $(\infty, \infty)$

d) domain $(-\infty, \infty)$, range $(\infty, \infty)$
Lesson 22-2:

A logarithm is an exponent to which a base is raised, that results in a specified value. A common logarithm is a base 10 logarithm.

---The Relationship---

\[ y = b^x \quad \text{(is equivalent to)} \quad \log_b(y) = x \]

(means the exact same thing as)

Example 1:

Which is a logarithmic statement for \(10^{3.5} = 3162.28\)?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( 3.5 \log 10 = 3162.28 )</td>
</tr>
<tr>
<td>B</td>
<td>( \log 3.5 = 3162.28 )</td>
</tr>
<tr>
<td>C</td>
<td>( 3162.28 \log 3.5 = 10 )</td>
</tr>
<tr>
<td>D</td>
<td>( \log 3162.28 = 3.5 )</td>
</tr>
</tbody>
</table>

Example 2:

Which is an exponential statement for \(\log 50 = 1.70\)?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( 1.70^{19} = 50 )</td>
</tr>
<tr>
<td>B</td>
<td>( 10^{1.70} = 50 )</td>
</tr>
<tr>
<td>C</td>
<td>( 10 (1.70)^x = 50 )</td>
</tr>
<tr>
<td>D</td>
<td>( 1.70 (10)^x = 50 )</td>
</tr>
</tbody>
</table>

Lessons 22-3 & 22-4:

Product Property of Logarithms: \( \log m + \log n = \log mn \)

Quotient Property of Logarithms: \( \log m - \log n = \log \frac{m}{n} \)

Product Property of Logarithms: \( \log m^n = n \log m \)
Unit Overview

In this unit, students study arithmetic and geometric sequences and implicit and explicit rules for defining them. Then they analyze exponential and logarithmic patterns and graphs as well as properties of logarithms. Finally, they solve exponential and logarithmic equations.

Student Focus

Main Ideas for success in lessons 23-1, 23-2, & 23-3

→ Extend concepts of logarithms to bases other than 10
→ Extend knowledge of inverse functions to include the inverse relationship between \( y = b^x \) and \( y = \log_b x \)
→ Discover and apply properties of logarithms
→ Apply the concept of graphing by transformations to logarithmic functions

Example:

Lesson 23-1:

Logarithms with bases other than 10 have the same properties as common logarithms.

The logarithm of \( y \) with base \( b \), where \( y > 0, b > 0, b \neq 1 \), is defined as: \( \log_b y = x \) if and only if \( y = b^x \).

Logarithms with base \( e \) are called \textit{natural logarithms}, and \( \log_e \) is written \( \ln \). So, \( \log_e x \) is written \( \ln x \).

Examples:

\textit{Inverse Property of Logarithms}:

What is \( 6^{\log_6 x} \)?

\textit{Answer:} \( x \)

What is equivalent to \( \log_7 a = b \)?

\textit{Answer:} \( 7^b = a \)

Lesson 23-2:

\textit{Change of Base Formula}:

\[ \log_b x = \frac{\log x}{\log b} \]

Example:

Given \( \log 15 = 1.1761 \) and \( \log 5 = 0.6990 \), find \( \log_5 15 \) rounded to four decimal places.

\textit{Answer:} Using change of base formula,

\[ \log_5 15 = \frac{\log 15}{\log 5} = \frac{1.1761}{0.6990} = 1.6825 \]
Lesson 23-3:

Asymptotes:
\[ f(x) = e^x \quad y = 0 \]
\[ g(x) = \ln x \quad x = 0 \]

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = e^x )</td>
<td>((-\infty, \infty))</td>
<td>((0, \infty))</td>
</tr>
<tr>
<td>( g(x) = \ln x )</td>
<td>((0, \infty))</td>
<td>((-\infty, \infty))</td>
</tr>
</tbody>
</table>
Unit Overview

In this unit, students study arithmetic and geometric sequences and implicit and explicit rules for defining them. Then they analyze exponential and logarithmic patterns and graphs as well as properties of logarithms. Finally, they solve exponential and logarithmic equations.

Student Focus

Main Ideas for success in lessons 24-1, 24-2, 24-3, & 24-4

- Explore exponential and logarithmic equations and solve them using properties of exponents and logarithms
- Use technology to approximate the solutions of exponential and logarithmic equations using tables of values and graphing
- Investigate and learn how to solve exponential and logarithmic inequalities

Example:

Example A

Solve $6 \cdot 4^x = 96.$

$6 \cdot 4^x = 96$

**Step 1:** $4^x = 16$

Divide both sides by 6.

**Step 2:** $4^x = 4^2$

Write both sides in terms of base 4.

**Step 3:** $x = 2$

If $b^m = b^n$, then $m = n$.

Example B

Solve $5^{4x} = 125^{x-1}$.

$5^{4x} = 125^{x-1}$

**Step 1:** $5^{4x} = (5^3)^{x-1}$

Write both sides in terms of base 5.

**Step 2:** $5^{4x} = 5^{3x-3}$

Power of a Power Property: $(a^m)^n = a^{mn}$

**Step 3:** $4x = 3x - 3$

If $b^m = b^n$, then $m = n$.

**Step 4:** $x = -3$ Solve for $x$. 
Lesson 24-2:

Example A

Estimate the solution of $3^x = 32$. Then solve to three decimal places.

Estimate that $x$ is between 3 and 4, because $3^3 = 27$ and $3^4 = 81$.

$3^x = 32$

**Step 1:** $\log_3 3^x = \log_3 32$

Take the log base 3 of both sides.

**Step 2:** $x = \log_3 32$

Use the Inverse Property to simplify the left side.

**Step 3:**

$x = \frac{\log 32}{\log 3}$

Use the Change of Base Formula.

**Step 4:**

$x \approx 3.155$

Use a calculator to simplify.

Example B

Find the solution of $4^{x-2} = 35.6$ to three decimal places.

$4^{x-2} = 35.6$

$\log_4 4^{x-2} = \log_4 35.6$

Take the log base 4 of both sides.

**Step 1:**

$x - 2 = \log_4 35.6$

Use the Inverse Property to simplify the left side.

**Step 2:**

$x = \log_4 35.6 + 2$

Solve for $x$.

**Step 3:**

$x = \frac{\log 35.6}{\log 4} + 2$

Use the Change of Base Formula.

**Step 4:**

$x \approx 4.577$

Use a calculator to simplify.
Example C

If Wesley deposits the gift from his grandfather into an account that pays 4% annual interest compounded quarterly, how much money will Wesley have in the account after three years?

Substitute into the compound interest formula. Use a calculator to simplify.

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 3000 \left( 1 + \frac{0.04}{4} \right)^{4(3)} \approx \$3380.48 \]

Solution: Wesley will have $3380.48 in the account after three years.

Example D

If Wesley deposits the gift from his grandfather into an account that pays 4% annual interest compounded continuously, how much money will Wesley have in the account after three years?

Substitute into the continuously compounded interest formula. Use a calculator to simplify.

\[ A = Pe^{rt} = 3000e^{0.04(3)} \approx \$3382.49 \]

Solution: Wesley will have $3382.49 in the account after three years.

Lesson 24-3:

Example A

Solve \( \log_4 (3x - 1) = 2 \).

\[ \log_4 (3x - 1) = 2 \]

**Step 1:**

\[ 4^{\log_4 (3x - 1)} = 4^2 \]

Write in exponential form using 4 as the base.

**Step 2:**

\[ 3x - 1 = 16 \]

Use the Inverse Property to simplify the left side.

**Step 3:**

\[ x = \frac{17}{3} \]

Solve for \( x \).

**Check:**

\[ \log_4 \left( 3 \cdot \frac{17}{3} - 1 \right) = \log_4 16 = 2 \]
**Example B**

Solve \( \log_3 (2x - 3) = \log_3 (x + 4) \).

\[
\log_3 (2x - 3) = \log_3 (x + 4)
\]

**Step 1:** \( 2x - 3 = x + 4 \)

If \( \log_b m = \log_b n \), then \( m = n \).

**Step 2:** \( x = 7 \)

Solve for \( x \).

**Check:** \( \log_3 (2 \cdot 7 - 3) \neq \log_3 (7 + 4) \)

\[
\log_3 11 = \log_3 11
\]

**Example C**

Solve \( \log_2 x + \log_2 (x + 2) = 3 \).

\[
\log_2 x + \log_2 (x + 2) = 3
\]

**Step 1:** \( \log_2 [x(x + 2)] = 3 \)

Product Property of Logarithms

**Step 2:** \( 2^{\frac{1}{2}}(x(x+2)) = 2^3 \)

Write in exponential form using 2 as the base.

**Step 3:** \( x(x + 2) = 8 \)

Use the Inverse Property to simplify.

**Step 4:** \( x^2 + 2x - 8 = 0 \)

Write as a quadratic equation.

**Step 5:** \( (x + 4)(x - 2) = 0 \)

Solve the quadratic equation.

**Step 6:** \( x = -4 \) or \( x = 2 \)

Check for extraneous solutions.

\[
\log_2 (-4) + \log(-4 + 2) \neq 3 \quad \log_2 2 + \log(2 + 2) \neq 3
\]

**Check:** \( \log_2 (-4) + \log(-2) \neq 3 \)

\[
\log_2 2 + \log 4 \neq 3
\]

\[
\log_2 8 \neq 3
\]

\[
3 = 3
\]

Because \( \log_2 (-4) \) and \( \log (-2) \) are not defined, \(-4\) is not a solution of the original equation; thus it is extraneous.

The solution is \( x = 2 \).
**Example D**

Solve $-x = \log x$ using a graphing calculator.

$-x = \log x$

![Graph showing intersection point (x=0.399, y=0.399)]

**Step 1:** Enter $-x$ for Y1.

**Step 2:** Enter $\log x$ for Y2.

**Step 3:** Graph both functions.

**Step 4:** Find the $x$-coordinate of the point of intersection: $x \approx 0.399$

**Solution:** $x \approx 0.399$

---

**Lesson 24-4:**

**Example A**

Use a graphing calculator to solve the inequality $4.2^{x+3} > 9$.

![Graph showing intersection point (x=-1.469, y=9)]

**Step 1:** Enter $4.2^{x+3}$ for Y1 and 9 for Y2.

**Step 2:** Find the $x$-coordinate of the point of intersection: $x \approx -1.469$

**Step 3:** The graph of $y = 4.2^{x+3}$ is above the graph of $y = 9$ when $x > -1.469$.

**Solution:** $x > -1.469$
Scientists have found a relationship between atmospheric pressure and altitudes up to 50 miles above sea level that can be modeled by \( P = 14.7(0.5)^{\frac{a}{35}} \). \( P \) is the atmospheric pressure in lb/in.\(^2\). Solve the equation \( P = 14.7(0.5)^{\frac{a}{35}} \) for \( a \). Use this equation to find the atmospheric pressure when the altitude is greater than 2 miles.

**Step 1:** Solve the equation for \( a \).

\[
\frac{P}{14.7} = 0.5^{\frac{a}{35}}
\]

Divide both sides by 14.7.

\[
\log_{0.5}\left(\frac{P}{14.7}\right) = \log_{0.5}\left(0.5^{\frac{a}{35}}\right)
\]

Take the log base 0.5 of each side.

\[
\log_{0.5}\left(\frac{P}{14.7}\right) = \frac{a}{35}
\]

Simplify.

\[
3.6\log_{0.5}\left(\frac{P}{14.7}\right) = a
\]

Multiply both sides by 3.6.

\[
\frac{3.6\log_{0.5}\left(\frac{P}{14.7}\right)}{\log_{0.5}} = a
\]

Use the Change of Base Formula.
**Step 2:** Use your graphing calculator to solve the inequality

\[
3.6\log\left(\frac{P}{14.7}\right) > 2.
\]

The graph of \( y = \frac{3.6\log\left(\frac{x}{14.7}\right)}{\log0.5} \) is above the graph of \( y = 2 \) when \( 0 < x < 10.002 \).

**Solution:** When the altitude is greater than 2, the atmospheric pressure is between 0 and 10.002 lb/in.².
LESSON 19-1

1. Which of the following is an arithmetic sequence?
   A. 2, 4, 8, 16, 32
   B. 1, 2, 3, 5, 8, 13
   C. 2, 4, 7, 11, 16, 22
   D. 6, 12, 18, 24, 30

2. What are the first five terms in each sequence?
   a. \(a_n = -2(n + 1)\)?
   b. \(a_n = (2n - 4)\)?
   c. \(a_n = 3(n - 2)\)?

3. What is the common difference in each sequence?
   a. -4, 1, 6, 11, 16
   b. 3, 7, 11, 15
   c. -12, 10, 32, 54

4. Construct viable arguments. Why is the sequence 2, 4, 8, 16, 32 not arithmetic?

5. Critique the reasoning of others. Amber says that the next two terms in the sequence 3, 7, 11, 15, 19 are 24 and 28. Is she correct? Why or why not?

7. Find the indicated partial sum of each arithmetic series.
   a. \(a_1 = 1, d = -2; \text{ find } S_{11}\)
   b. \(4, 15, 26, 37 \ldots; \text{ find } S_{14}\)
   c. \(-17, -8, 1, 10 \ldots; \text{ find } S_{15}\)

8. Model with mathematics. Brian just posted a social media story. If he shares the post with 11 friends when he posts, and 5 new people see the post each minute, how many total people will have seen it after:
   a. \(\frac{1}{2}\) hour?
   b. 45 minutes?
   c. 55 minutes?

9. Attend to precision. What is the sum of the first 11 numbers in the arithmetic sequence where \(a_1 = 1.3\) and \(d = 0.65\)?

10. Find the indicated term in each sequence.
    a. \(a_1 = 5, d = 7; \text{ find } a_6\)
    b. \(a_1 = -9, d = -4; \text{ find } a_{13}\)
    c. \(a_1 = 24, d = 9; \text{ find } a_{17}\)

LESSON 19-2

6. Which is the partial sum \(S_n\), given \(a_1 = 6\) and \(d = 9\)?
   A. 15        B. 72
   C. 300       D. 432

11. Which is the correct value of the partial sum
    \(\sum_{n=2}^{12} 1 + \frac{n-1}{2} + \sum_{n=-9}^{9} 3 + (2n-2)\)?
    A. 21        B. 63
    C. -57       D. 432
12. Which is the correct value of the partial sum \[ \sum_{n=-3}^{9} -8 + \frac{3n-3}{4} - \sum_{n=-10}^{7} 5 + \frac{5n-5}{4} \]?
   A. \(-50.75\)   B. \(-118.25\)
   C. 43   D. 118.25

13. **Attend to precision.** Evaluate:
   \[ \sum_{n=8}^{13} 1 - \frac{n-1}{2} \]

14. Evaluate: \[ \sum_{n=-1}^{0} 5 - 3(n-1) \]

15. **Make use of structure.** Evaluate:
   \[ \sum_{n=0}^{10} (n+1) - \sum_{n=1}^{11} n + 1 \]

**LESSON 20-1**

Identify whether the sequence is arithmetic, geometric, or neither. If arithmetic, state the common difference. If geometric, state the common ratio.

16. 11, 22, 34, 47, 61 …

17. 17, 51, 153, 459, …

18. Identify the 8th term in the geometric sequence: \( a_1 = 3, \ r = 4 \).

19. **Construct viable arguments.** True or False: If the first three terms of a geometric sequence are \( \frac{1}{4}, \ \frac{1}{2}, \) and 9, then \( r = 9 \). Explain your answer.

20. **Express regularity in repeated reasoning.** After a ball is dropped, it bounces to a height of 9.2 meters on the first bounce. Assuming the ball bounces to 80% of its previous height on each bounce, determine the maximum height of the ball after the fourth bounce.

**LESSON 20-2**

21. Which is the sum of the first 6 terms of the geometric series? \( 7 + 21 + 63 \ldots \)
   A. 630   B. 1701
   C. 2548   D. 3780

22. To the nearest hundredth, which is the partial sum \( S_{12} \), given \(-2 + -5 + -12.5 + -31.25 + \ldots S_{12} \)?
   A. \(-1231.13\)   B. \(-2462.26\)
   C. 27,512.25   D. \(-79,471.53\)

23. **Express regularity in repeated reasoning.** Evaluate the following:
   a. \( \sum_{n=1}^{8} 5(-2)^{n-1} \)
   b. \( \sum_{x=1}^{7} -8(4)^{x-1} \)
   c. \( \sum_{j=1}^{3} 5(0.75)^{j-1} \)
   d. \( \sum_{j=1}^{4} 9(-3)^{j-1} \)

24. **Make sense of problems.** A flash mob occurs when an unexpectedly large group of people suddenly appears in a single location with little or no notice. Commonly, this is arranged by text messages. Suppose five friends decide to start a flash mob, and each of them sends a text to five more, who each send it to five more, and so on:
   a. Write the rule that defines the number of texts that go out during the \( n \)th iteration.
   b. If the mob includes everyone who sent or received a text over five rounds of messages, how many people are at the location?

25. How many terms are in the series defined by \( 3 + 1.5 + 0.75 + \ldots 0.0234 \)?
LESSON 20-3

26. Which is the correct value of the infinite sum
\[ \sum_{n=1}^\infty -9 \left( \frac{-1}{3} \right)^{n-1} \]?
A. -27  B. -4  C. -6.75  D. 13.5

27. Find the infinite sum if it exists. If it does not exist, explain why not.

a. \[ 14 + 7 + 3.5 + 1.75 + \ldots \]
b. \[ 1.25 + 1.875 + 2.8125 + \ldots \]
c. \[ 2 + 2 + 2 + \ldots \]
d. \[ 5000 + 4750 + 4512.50 + \ldots \]

28. Determine whether each series converges or diverges.

a. \[ 1 + -2 + 4 + -8 \ldots \]
b. \[ -8 + 4 + -2 + 1 \ldots \]
c. \[ 0.001 + 0.005 + 0.025 + \ldots \]
d. \[ 0.01 + 0.009 + 0.008 + 0.007 + \ldots \]

29. Reason quantitatively. An infinite geometric series has a first term of 5 and a sum of 6.66. Find r.

30. Critique the reasoning of others. Tuscany is complaining about having to vacuum her room before she can go out with her friends. Her mom is a math teacher and suggests she break the job up into sections to make it go faster. Tuscany says that won’t work because if she vacuums half of the room in twenty minutes, and half of the remainder in ten more and so on, she will never finish, since there will always be some portion remaining if she maintains the same pace. Tuscany’s mom says that is nonsense; she will finish in 40 minutes. Who is correct? Defend your answer.

LESSON 21-1

31. Reason abstractly. A function is described as follows: “Take the current value and subtract 1.42 to get the next value.” If a graph connects all of the values of the function, what form will it take?

32. Express regularity in repeated reasoning.

a. Complete the table so that the described function is linear.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>2.7</td>
<td>4.5</td>
<td></td>
<td></td>
<td>9.9</td>
</tr>
</tbody>
</table>

b. What is the function that describes the data?

33. Which function described by the ordered pairs is linear?
A. (1, 5.2), (6, 31.2), (9, 46.8)
B. (3, 6), (5, 12), (9, 25)
C. (1, 2), (2, 4), (4, 5)
D. (0.25, 1.5), (0.50, 3.0), (0.75, 4.25)

34. Determine if each function described by the ordered pairs is exponential.

a. (1, 5), (2, 3.75), (3, 2.81)
b. (5, 10), (6, 100), (7, 1000)
c. (1, 13), (3, 4.68), (6, 1.01)

35. Which function results in the greatest value, given \( x = 5 \)?
A. \( y = 3x + 50 \)
B. \( y = 3^x + 50 \)
C. \( y = 3^{50-x} \)
D. \( y = 50^x + 3 \)
**LESSON 21-2**

36. Which statement is incorrect, given \(f(x) = 2(1.25)^x\)?
   - A. The exponential growth factor is 125%.
   - B. The percent of increase is 25%.
   - C. The scale factor is 0.25.
   - D. The rate of growth is 25%.

37. Which statement is correct, given \(f(x) = 2(1.25)^x\)?
   - A. \(a = 2, b = 1.25\)
   - B. \(a = 2.5, b = 1.25^x\)
   - C. \(a = 1.25, b = 2\)
   - D. \(a = 2.5^x, b = 2\)

38. **Reason quantitatively.** Determine whether the data in the table can be modeled by a linear function or an exponential function and write the function. Write “neither” if the data are not linear or exponential.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{9}{16})</td>
</tr>
</tbody>
</table>

39. **Make sense of problems.** Given the function \(f(x) = 90 \left(\frac{1}{3}\right)^x\), complete the following.
   - a. Determine the rate of decay.
   - b. Determine the decay factor.
   - c. If the function describes the money left in Tony’s wallet after each of a series of purchases, what is the reasonable domain and range of the function? Why?

40. Write the exponential function described by \(a = 7.5, b = \frac{1}{3}\). Is this a growth or decay function?

**LESSON 21-3**

41. **Reason abstractly.** Show mathematically that the graph of \(f(x) = \left(\frac{1}{3}\right)^x\) is the same as the graph of \(f(x) = 3^{-x}\).

42. **Model with mathematics.** What type of function is modeled by the graph?

   ![Graph](image)

43. Which function describes the graph in Item 42?
   - A. \(f(x) = 3(2x)\)
   - B. \(f(x) = 3 + (2x)\)
   - C. \(f(x) = 3(x^2)\)
   - D. \(f(x) = 3(2)^x\)
44. For each exponential function, state the domain and range, whether the function increases or decreases, and the $y$-intercept.
   a. $f(x) = 3(5)^x$
   b. $f(x) = 5 \left(\frac{1}{3}\right)^x$
   c. $f(x) = -2(0.25)^x$
   d. $f(x) = -(3.9)^x$

45. Under what conditions is the function $f(x) = a(2)^x$ decreasing?

LESSON 21-4
46. Reason abstractly. What single transformation can be used to map the function $f(x) = 4^x$ to $g(x) = -4^x$?

47. Use appropriate tools strategically. Given the function $g(x) = 2(3)^{-x} + 2$, complete the following.
   a. Use technology to graph the function on the same graph as the parent $f(x) = 3^x$.
   b. What is the equation of the asymptote of $g(x) = 2(3)^{-x} + 2$?
   c. Describe the transformation(s) involved with mapping the parent function to the function $g(x)$.

48. What transformation results from substituting $x + 4$ for $x$ in the function $f(x) = 2^x$?

49. What transformation results from adding 4 to $2^x$ in the function $f(x) = 2^x$?

50. Reason abstractly. Under what conditions does an increase in the value of $a$ in the function $f(x) = a(2)^x$ result in a vertical compression rather than a vertical stretch?

LESSON 21-5
51. Which function has a $y$-intercept of 1?
   A. $f(x) = (e + 1)^x$
   B. $f(x) = (e)^{x + 1}$
   C. $f(x) = e^x$
   D. $f(x) = e^x + 1$

52. Which function outputs the greatest value for $f(-4)$?
   A. $f(x) = e^x$
   B. $f(x) = e^{-x}$
   C. $f(x) = e + x$
   D. $f(x) = xe$

53. Reason quantitatively. Between which two integer values of $a$ in the function $f(x) = a^x$ is the function $f(x) = e^x$?

54. Use appropriate tools strategically. Given the function $g(x) = 3(e^x) - 2$, complete the following.
   a. Use technology to graph $g(x)$ on the same graph as $f(x) = e^x$.
   b. Describe the transformation(s) to map $f(x)$ to $g(x)$.
   c. State the domain, range, and asymptote of $g(x)$.

55. Write the function $g(x)$, given that $g(x)$ may be graphed by transforming the parent function $f(x) = e^x$ as follows: a vertical stretch by 2.8, reflection over the $x$-axis, and a horizontal shift to the right by 4.2.

LESSON 22-1
56. Reason quantitatively. How does the ground motion caused by a magnitude 5.0 earthquake compare to that of a magnitude 8.0 earthquake?
57. **Reason abstractly.** Is an increase in Richter Scale magnitude of 2.0 from 6.0 to 8.0 more dangerous than an increase of 3.0 from 1.0 to 4.0? Defend your answer.

58. Does a magnitude 0 earthquake indicate no ground motion? Explain.

59. Use the function \( G(x) = 10^x \) to determine how many times as great the ground motion of earthquakes of the following magnitudes are compared to a magnitude 0 earthquake.
   a. magnitude 2.8
   b. magnitude 3.6
   c. magnitude 4.7
   d. magnitude 7.1

60. A certain earthquake causes about 35,000 times as much ground motion as a magnitude 0 quake. Using \( G(x) = 10^x \), between which two integer values of \( x \) is the Richter Scale magnitude of this earthquake?

**LESSON 22-2**

61. **Use appropriate tools strategically.** The common logarithmic function \( M(x) = \log x \) may be used to estimate the Richter Scale rating of an earthquake with \( x \) times the motion of a magnitude 0 quake. Use this function with technology to estimate to the nearest tenth the Richter Scale rating of earthquakes with the following multiples of magnitude 0 motion.
   a. 2000 times
   b. 25,000 times
   c. 45,000,000 times
   d. 6,000,000,000 times

62. Write an equivalent logarithmic statement for each exponential statement.
   a. \( 10^3 = 1000 \)
   b. \( 10^6 = 1,000,000 \)
   c. \( 10^{-2} = \frac{1}{100} \)
   d. \( 10^{-4} = \frac{1}{10,000} \)

63. **Make sense of problems.** Write an equivalent exponential statement for each logarithmic statement.
   a. \( \log 10,000 = 4 \)
   b. \( \log 100 = 2 \)
   c. \( \log \frac{1}{10,000} = -4 \)
   d. \( \log \frac{1}{10,000,000} = -7 \)

64. Evaluate.
   a. If \( \log n = m \), what does \( 10^m \) equal?
   b. If \( 10^b = a \), what does \( \log a \) equal?

65. If \( \log a = x \) and \( 1000 < a < 100,000 \), which is an appropriate range of values for \( x \)?
   A. \( 1000 < x < 100,000 \)
   B. \( 10 < x < 100 \)
   C. \( 1 < x < 10 \)
   D. \( 3 < x < 6 \)

**LESSON 22-3**

66. **Make use of structure.** Expand each expression:
   a. \( \log 5x \)
   b. \( \log \frac{3}{y} \)
   c. \( \log x^4 \)
   d. \( \log \frac{m^3}{4} \)
67. **Attend to precision.** Given \( \log 7 = 0.845 \) and \( \log 11 = 1.04 \), evaluate:

   a. \( \log \frac{7}{11} \)
   
   b. \( \log \frac{11}{7} \)
   
   c. \( \log 77 \)

68. Rewrite each expression as a single logarithm:

   a. \( \log 4 + \log y - (\log 7 + \log x) \)
   
   b. \( 5 \log x - 2 \log y \)
   
   c. \( \log x + \log 4 - (3 \log y + \log 4) \)
   
   d. \( \log a - \log b \)

69. Which expression is *not* equal to 5?

   A. \( \log 10^5 \)
   
   B. \( \log \left( \frac{10^{12}}{10^7} \right) \)
   
   C. \( \frac{\log 10^9}{\log 10^5} \)
   
   D. \( \log 10^9 - \log 10^4 \)

70. Given \( \log 13 = 1.1 \) and \( \log 5 = 0.7 \), what is \( \log 65 \)? Show your work and do not use technology.

**LESSON 22-4**

71. **Construct viable arguments.** Is \( \log xy^2 \) equal to \( 2(\log x - \log y) \)? Explain your answer and state any relevant logarithm properties.

72. **Reason abstractly.** If \( \log 10 = \log 2 + \log 5 \), does it stand to reason that \( \log mn = \log m + \log n \)? Explain your answer.

73. Expand \( \log \frac{u^2}{y^3} \) and state the properties used to do so.

74. Rewrite each expression as a single logarithm and evaluate. Do not use a calculator.

   a. \( \log 200 + \log 50 \)
   
   b. \( 3 \log 5 + (\log 2 - \log 250) \)
   
   c. \( \log 250 - 2 \log 5 \)

75. Which expression is equal to \( \log \left( \frac{3x^2 + y^2}{2x^3} \right) \)?

   A. \( 3 \log x + \log 4y + \log 2 - (3 \log 2x) \)
   
   B. \( 4 \log x + 2 \log y + \log 3 - (3 \log 2x) \)
   
   C. \( 4 \log x + 2 \log y + \log 3 - (\log 2 + 3 \log x) \)
   
   D. \( \log 3 + \log 4x + 2 \log y - (\log 2 + \log 3x) \)

**LESSON 23-1**

76. Given \( f(x) = 3^x \), which function represents \( f^{-1}(x) \)?

   A. \( 3 \log x \)  
   
   B. \( \log 3x \)
   
   C. \( \log -3x \)  
   
   D. \( \log x \)

77. **Make use of structure.** Simplify each expression.

   a. \( 4^{\log_4 x} \)
   
   b. \( \log_5 5^x \)
   
   c. \( 8^{\log_8 y} \)

78. **Make sense of problems.** Given \( f(x) = 3x - 4 \), write the inverse function \( f^{-1}(x) \). Show your work.

79. What is the line of symmetry between \( f(x) = 2x - 3 \) and its inverse function \( f^{-1}(x) = x + 3 \)? Explain.
80. Let \( g(x) = f^{-1}(x) \), the inverse of function \( f \). Write the rule for \( g(x) \) for each function \( f \).

a. \( f(x) = \left( \frac{1}{4} \right)^x \)

b. \( f(x) = 5x - 2 \)

c. \( f(x) = -2x + 7 \)

d. \( f(x) = 5^x \)

e. \( f(x) = \ln x \)

LESSON 23-2

81. Make use of structure. Express each exponential statement as a logarithmic statement.

a. \( 5^3 = 125 \)

b. \( 7^{-3} = \frac{1}{343} \)

c. \( 10^0 = 1 \)

d. \( 4^3 = 64 \)

82. Make use of structure. Express each logarithmic statement as an exponential statement.

a. \( \log_3 81 = 4 \)

b. \( \log_7 729 = 3 \)

c. \( \ln 1 = 0 \)

d. \( \log_6 \left( \frac{1}{216} \right) = -3 \)

83. Evaluate \( \log_5 625 \) without a calculator.

84. Evaluate \( \log_{1024} 1 \) without a calculator. Explain your answer.

85. Which expression is equivalent to \( 2 \ln 4 + \ln y? \)

A. \( \ln 16y \)

B. \( 2 \ln y \)

C. \( \ln 4y^2 \)

D. \( 2 \ln 4y \)

LESSON 23-3

86. Express regularity in repeated reasoning. Complete the table for \( f(x) = 3^x \).

\[
\begin{array}{c|cccc}
  x & -2 & -1 & 0 & 1 \\
  f(x) = 3^x & \frac{1}{9} & \frac{1}{3} & 1 & \frac{3}{3} \\
\end{array}
\]

87. Express regularity in repeated reasoning. Complete the table for \( g(x) = \log_3 x \).

\[
\begin{array}{c|cccc}
  x & \frac{1}{9} & \frac{1}{3} & 1 & 9 \\
  g(x) = \log_3 x & & & & \\
\end{array}
\]

88. Graph both \( f(x) = 3^x \) and \( g(x) = \log_3 x \) on the same coordinate plane. Identify the \( x- \) and \( y- \)intercepts of each, and sketch the line of symmetry.

89. State the domain and range of each function in Item 88 using interval notation.

90. What is the equation of the asymptote for \( g(x) = \log_3 x \)?

A. \( y = 0 \)

B. \( x = 0 \)

C. \( y = x \)

D. \( x = 3 \)
Lesson 24-1

91. Which equation can be solved by rewriting both sides in terms of the same base?
   A. \(3^x = 15\)  
   B. \(4^{x-2} + 3 = 20\)  
   C. \(2^{x-5} - 4 = 12\)  
   D. \(5^x = 20\)

92. Express regularity in repeated reasoning. Solve for \(x\).
   a. \(36^x = 1296\)  
   b. \(144^x = 20,736\)  
   c. \(125^x = 625\)  
   d. \(343^x = 49\)

93. Express regularity in repeated reasoning. Solve for \(x\).
   a. \(256^{-x+2} = 2\)  
   b. \(9^{2x-1} = 27^2\)  
   c. \(9^{-4x+5} + 7 = 88\)  
   d. \(81^{x-1} + 2 = 8 + 3\)

94. Solve for \(x\): \(e^{5x} = 3\).

95. Solve for \(x\): \(2e^{3x} = 52\).

Lesson 24-2

96. Between which two integers is the value of \(x\) in the equation \(3^x = 271\)?
   A. 2 and 3  
   B. 3 and 4  
   C. 4 and 5  
   D. 5 and 6

97. Between which two integers is the value of \(x\) in the equation \(6^{x+1} = 1325\)?
   A. 2 and 3  
   B. 3 and 4  
   C. 4 and 5  
   D. 5 and 6

Lesson 24-3

98. Reason quantitatively. Estimate the value of \(x\) to the nearest integer. Do not use a calculator.
   a. \(2^x = 15\)  
   b. \(4^x = 81\)  
   c. \(12^x = 211\)  
   d. \(7^x = 289\)

99. Use appropriate tools strategically. Solve for \(x\) to three decimal places. Use technology to simplify once you have isolated the variable.
   a. \(2^x = 15\)  
   b. \(4^x = 81\)  
   c. \(12^x = 211\)  
   d. \(7^x = 289\)

100. To three decimal places, what is the value of \(x\), given \(5^{x+2} = 148\)?

101. Which rule applies to solving \(\log_4 (3x - 7) = \log_4 (x + 5)\) for \(x\)?
    A. \(\log_b m = \log_b n\) if and only if \(m = n\)  
    B. \(\log_b a = c\) if and only if \(b^c = a\)  
    C. \(\log_10 x = \log x\)  
    D. \(\ln x = \log_e x\)

102. Which rule applies to solving \(\log_4 (2x + 4) = 2\) for \(x\)?
    A. \(\log_b m = \log_b n\) if and only if \(m = n\)  
    B. \(\log_b a = c\) if and only if \(b^c = a\)  
    C. \(\log_10 x = \log x\)  
    D. \(\ln x = \log_e x\)
103. Make use of structure. Solve for $x$.
   a. $\log_4 (3x - 7) = \log_4 (x + 5)$
   b. $\log_e (8x + 1) = \log_e (2x - 3)$
   c. $\log (2x + 7) = \log x$
   d. $\ln (2x + 3) = \log_e (5x + 21)$

104. Make use of structure. Solve for $x$.
   a. $\log_4 2x - \log_4 6 = \log_4 3$
   b. $\log_e (5x + 3) - \log_e 2 = \log_e (3x + 4)$
   c. $\log (x + 3) + \log 4 = \log (x - 2)$
   d. $\ln (7x - 1) + \ln 3 = \log_e (2x - 16)$

105. State the domain of each equation using interval notation.
   a. $f(x) = \log (x - 4)$
   b. $f(x) = \log (x + 5)$
   c. $f(x) = \log (2x - 9)$
   d. $f(x) = \log (3x + 2)$

LESSON 24-4

106. Which is the correct solution set for $\log 10x \geq 1.3$?
   A. $x \geq 3.162$    B. $x \geq 1.995$
   C. $x \geq 1.487$    D. $x \geq 5.972$

107. Attend to precision. Using technology, solve for $x$ to three decimal places.
   a. $\log 2x \geq 2.9$
   b. $\log (3x - 5) \leq 1.6$
   c. $4.2 \log 3x - 3.8 \geq -2.9$

108. Use appropriate tools strategically. Use technology to generate a graph and find the solution set for $\log 7x + 2 \leq 3.5$.

109. To three decimal places, what is the solution set of $x$, given $\log_2 (2x + 4) \geq \log_2 (x - 1.41)$?

110. To three decimal places, what is the solution set of $x$, given $\log_2 (3x - 7) \leq \log_4 21$?