Unit Overview

In this unit, students will study formal definitions of basic figures, the axiomatic system of geometry and the basics of logical reasoning. They will also write equations of parallel and perpendicular lines.

Student Focus

Main Ideas for success in lessons 24-1, 24-2, and 24-3:

→ Describe relationships among tangents and radii of a circle.
→ Use arcs, chords, and diameters of a circle to solve problems.
→ Describe relationships among diameters and chords of a circle.
→ Prove and apply theorems about chords of a circle.
→ Prove that tangent segments to a circle from a point outside the circle are congruent.
→ Use tangent segments to solve problems.

Example

Lesson 24-1:

Given that $\overline{OT} = 5$, $\overline{XT} = 12$, and $\overline{OX} = 13.6$, is $\overline{XY}$ tangent to circle $O$ at $T$? Explain your reasoning.

→ Yes, $\overline{XY}$ is perpendicular to the radius $\overline{OT}$, because

$$13.6^2 = 12^2 + 5^2$$
Example

Lesson 24-2:

The diameter of a circle with center P bisects $AB$ at point X. Points A and B lie on the circle. Classify the triangle formed by points A, X, and P.

$\rightarrow$ Right Triangle

Example

Lesson 24-3:

In the diagram, $NM$ and $QN$ are tangent to a circle P, the radius of circle P is 5 cm, and $MN = 12$ cm.

What is $QN$?

$\rightarrow$ 12 cm

What is $PN$?

$\rightarrow$ 13 cm
Unit Overview

In this unit, students will study formal definitions of basic figures, the axiomatic system of geometry and the basics of logical reasoning. They will also write equations of parallel and perpendicular lines.

Student Focus

Main Ideas for success in lessons 25-1, 25-2, 25-3, and 25-4:

→ Understand how to measure an arc of a circle.
→ Use relationships among arcs and central angle to solve problems.
→ Describe the relationship among inscribed angles, central angles, and arcs.
→ Use inscribed angles to solve problems.
→ Describe a relationship among the angles formed by intersecting chords in a circle.
→ Use angles formed by chords to solve problems.
→ Describe relationships among the angles formed by tangents to a circle or secants to a circle.
→ Use angles formed by tangents or secants to solve problems.
**Example**

**Lesson 25-1:**

Use the circle shown below to determine the following.

![Diagram of a circle with 96° and 8x angle]

What is the measure of the central angle?

→ 96°

Write and solve an equation for x.

→ 96 = 8(x); x = 12

What is the measure of the angle that bisects the minor arc?

→ 48°

Write the expression that can be used to determine the measure of the major arc.

→ 360° - 96°
Lesson 25-2:

Use the diagram (not drawn to scale) below to answer the following items.

a) Which angle(s), if any, are right angles?
   
   → \( \angle ADB \)
   
   b) What is the sum of \( m\angle DAB \) and \( m\angle DCB \)? Explain.
   
   → 180°. Opposite angles of a quadrilateral inscribed in a circle are supplementary.
   
   c) Identify the angle with a measure of 40°.
   
   → \( \angle DAB \)
**Example**

**Lesson 25-3:**

Write an expression to determine the value of \( x \).

\[
\rightarrow x = \frac{1}{2}(50 + 20)
\]

**Example**

**Lesson 25-4:**

Determine \( m\angle 2 \), if \( m\angle 1 = 34^\circ \)

\[
\rightarrow m\angle 2 = 73^\circ
\]
Unit Overview

In this unit, students will study formal definitions of basic figures, the axiomatic system of geometry and the basics of logical reasoning. They will also write equations of parallel and perpendicular lines.

Student Focus

Main Ideas for success in lessons 27-1 and 27-2:

- Derive the general equation of a circle given the center and radius.
- Write the equation of a circle given three points on the circle.
- Find the center and radius of a circle given its equation.
- Complete the square to write the equation of a circle in the form
  
  \[(x - h)^2 + (y - k)^2 = r^2\]

Unit 4 Vocabulary

Tangent Lines  
Radius  
Diameter  
Chord  
Arc  
Conjecture  
Equidistant  
Bisecting Ray  
Minor Arc  
Major Arc  
Central Angle  
Inscribed Angle  
Interior  
Half  
Vertical Angle  
Radii  
Tangent  
Completing the Square
Example

Lesson 27-1:

Write the equation of a circle given the center and radius.

a) Center: (7,2); radius = 5
   \[ (x - 7)^2 + (y - 2)^2 = 25 \]

b) Center: (-4,-2); radius = 9
   \[ (x + 4)^2 + (y + 2)^2 = 81 \]

Example

Lesson 27-2:

Determine if the equation is representative of a circle. If so, determine the coordinates of the center of the circle.

\[ x^2 - 2x + y^2 + 10y = 0 \]

\[ \rightarrow \text{Yes; (1, -5)} \]
Unit Overview

In this unit, students will study formal definitions of basic figures, the axiomatic system of geometry and the basics of logical reasoning. They will also write equations of parallel and perpendicular lines.

Student Focus

Main Ideas for success in lessons 29-1, 29-2, and 29-3:

→ Use construction to copy a segment or an angle.
→ Use construction to bisect a segment or an angle.
→ Construct parallel and perpendicular lines.
→ Use constructions to make conjectures about geometric relationships.
→ Construct inscribed and circumscribed circles.
→ Construct tangents to a circle.
Example

Lesson 29-1:
Describe a way to divide $\overline{WO}$ into four congruent segments.

$\overline{WO}$

→ Construct the perpendicular bisector of $\overline{WO}$. Then construct the perpendicular bisector of each of the two segments that are formed.

Example

Lesson 29-2:
Construct and label a right triangle with legs of lengths HI and JK. (Hint: Extend $\overline{JK}$ and construct a perpendicular line at point J. Then construct a segment congruent HI on the perpendicular line.)

$\overline{HI}$

$\overline{JK}$

$\overline{UT}$
Example

Lesson 29-3:

Explain the steps you would use to construct a tangent to circle $C$ at a point $P$.

→ First use the straightedge to draw $\overrightarrow{CP}$. Then construct the perpendicular line to $\overrightarrow{CP}$ that passes through point $P$. The perpendicular line will be tangent to circle $C$. 
LESSON 24-1

1. Use the diagram shown.

![Diagram with labeled points A, B, C, D, E, F, G, H]

a. Identify a line that is tangent to the circle.

b. Identify a radius of the circle.

c. Identify a chord of the circle.

d. Which segment is perpendicular to the tangent line?

2. Point A is a point on circle O. Which statement is NOT true?

A. $\overline{AO}$ is a radius of the circle.

B. There are many chords of the circle that contain point A.

C. There are many tangent lines that contain point A.

D. There is exactly one diameter that contains point A.

3. In the diagram shown, $\overline{TA}$ is tangent to circle $P$, the radius of the circle is 7 units, and $TA = 24$ units. Find $TB$.

![Diagram with labeled points T, A, P, B]

4. Attend to precision. Line $WX$ is tangent to circle $T$ at point $X$. Line $WT$ intersects the circle at points $P$ and $Q$. The radius of circle $T$ is 9 units and $WP = 6$ units. What is $WX$?

![Diagram with labeled points W, X, T, P, Q]

5. Reason quantitatively. In this diagram, the radius of circle $M$ is 10 and $TS = SQ = 8$. What is the length $SN$?

![Diagram with labeled points N, Q, S, T, M, P]
LESSON 24-2

6. Make use of structure. The distance between the center of a circle and a chord is 15 cm.
   a. If the radius of the circle is 20 cm, what is the length of the chord?

   b. If the length of the chord is 20 cm, what is the radius of the circle?

7. The length of a chord of a circle is 21.2 cm and that chord is 5 cm from the center of the circle.
   a. What is the length of the diameter of the circle?

   b. What is the length of a chord of the circle that is 3 cm from the center of the circle?

8. Think about a chord and a diameter of the same circle.
   a. How are they similar?

   b. How can they be different?

9. Which of the following statements is NOT true?
   A. If a radius is perpendicular to a chord, it bisects the chord.
   B. If two chords are perpendicular, one of them must bisect the other.
   C. If a diameter is perpendicular to a chord, it bisects the chord.
   D. Any two distinct diameters of a circle bisect each other.

10. Reason quantitatively. In the diagram, chords \( PQ \) and \( RS \) intersect at point \( T \).

    a. If \( PT = 8 \), \( PQ = 3 \), and \( RT = 6 \), what is \( ST \)?

    b. If \( PQ = 12 \), \( TQ = 2 \), and \( RT = 5 \), what is \( ST \)?

    c. If \( PT = m \), \( TQ = n \), and \( ST = p \), what is \( RT \) in terms of \( m \), \( n \), and \( p \)?

    d. If \( RT = ST = 5\sqrt{2} \) and \( PT = 2 \cdot QT \), what is \( PQ \)?

LESSON 24-3

11. In the diagram shown, \( AM \) and \( AN \) are tangent to circle \( R \).

    a. If \( MP = 16 \) and \( AM = 15 \), what is \( QA \)?

    b. If \( MP = 24 \) and \( QA = 8 \), what is \( AN \)?

    c. If \( AQ = a \) and \( RQ = b \), write an expression for \( AN \) in terms of \( a \) and \( b \).

    d. If \( AM = 12 \) and \( QA = 8 \), what is \( MP \)?
12. In the diagram shown $AB = AC$, $AB = 15$, $CP = 4$ and $AB$, $BC$, and $AC$ are tangent to circle $X$.

[Diagram of triangle ABC with a circle inside and points Q, P, and X marked on the circle]

a. Find the perimeter of $\triangle ABC$.

b. If the radius of circle $X$ is 2 units, what is $BX$? Write your answer as a radical.

13. Which statement about tangents to a circle is NOT true?

A. A tangent to a circle is perpendicular to a radius at the point of tangency.

B. If two segments are tangent to a circle from the same point $A$ outside the circle, the ray from $A$ to the center of the circle bisects the angle formed by the two tangents.

C. If two segments from the same point outside a circle are tangent to the circle, then the line joining the two points of tangency can be a diameter of the circle.

D. If $AB$ is tangent to circle $O$ at point $P$, and $Q$ is any point on $B$ other than $P$, then there is another line through $Q$ that is tangent to circle $O$.

14. Construct viable arguments. This diagram can be used to prove the theorem about two tangents to a circle from a point outside the circle.

[Diagram of circle with points labeled R, S, T, M, N, Q, and X]

a. How are the sides of $RSTQ$ related to each other?

b. What kind of figure is $RSTQ$? Explain.

c. How are angles $TSR$ and $TQR$ related to each other? Explain.

d. How are angles $STQ$ and $SRQ$ related to each other? Explain.

e. How are segments $SQ$ and $TR$ related to each other?

15. Make sense of problems. In the diagram, the three segments are tangent to the circle. $DE = 17$, $DF = 12$, and $DH = 9.5$.

[Diagram of triangle DEF with a circle inside and points I, J, K, H, G, and F marked on the circle]

a. Find the perimeter of $\triangle DEF$.

b. If the radius of the circle is 3, find the distance from the center of the circle to point $E$ to the nearest tenth.
LESSON 25-1

16. Make use of structure. In circle $T$, $m\angle BTD = 60^\circ$, $TC$ bisects $\angle BTD$, and $AB$ is a diameter.

![Diagram of circle T with points A, B, C, and D]

a. What is $m\angle ATC$?

b. What is $m\angle CTD$?

c. Identify three major arcs.

d. Name three adjacent arcs that form a semicircle.

17. In circle $D$, $m\angle PDR = 38^\circ$. Find each measure.

![Diagram of circle D with points P, R, D, Q, and S]

a. $m\widehat{PR}$

b. $m\widehat{PRS}$

c. $m\angle SPQ$

d. $m\angle QDR$

e. $m\widehat{SQ}$

18. Which of the following statements is true?

A. The two radii that form a major arc can also form a diameter.

B. A minor arc, plus a major arc, can form a full circle.

C. The total measure of a major arc and a minor arc can be $180^\circ$.

D. A major arc and a minor arc can form a semicircle.

19. In the diagram of circle $P$ with diameter $AB$, $m\widehat{CB} = (5x - 7)^\circ$ and $m\widehat{AC} = 12x^\circ$.

![Diagram of circle P with points A, B, C, and P]

a. Find $x$.

b. Find $m\angle APC$ and $m\angle CPB$.

20. Model with mathematics. In the diagram shown, $\overline{TA}$ and $\overline{TB}$ are tangent to circle $C$ at points $A$ and $B$. The measure of $\angle ATB$ is $36^\circ$, and $P$ is a point on major arc $APB$.

![Diagram of circle C with points A, B, C, D, and T]

a. Find $m\angle ACB$.

b. Find $m\widehat{AD}$.

c. Find $m\widehat{AB}$.

d. Find $m\widehat{APB}$.
**LESSON 25-2**

21. **Reason quantitatively.** In circle $P$, $\overline{AB}$ and $\overline{CD}$ are diameters, $m\angle CDE = 28^\circ$, and $m\overline{AD} = 90^\circ$. Find each measure.

![Diagram of circle with labeled segments A, B, C, D, E, and P.]

- **a.** $m\overline{EC}$
- **b.** $m\angle ACB$
- **c.** $m\angle ACD$
- **d.** $m\overline{CB}$
- **e.** $m\angle CBA$

22. In the diagram shown, $\overline{PS}$ is a diameter of circle $A$, $m\overline{RS} = 40^\circ$, $m\overline{PQ} = 85^\circ$, and $m\overline{PT} = 129^\circ$. Find each measure.

![Diagram of circle with labeled segments P, Q, R, S, and T.]

- **a.** $m\angle RPS$
- **b.** $m\angle QPR$
- **c.** $m\angle SPT$
- **d.** $m\overline{QST}$
- **e.** $m\overline{QPT}$

23. A square is inscribed in a circle. If the area of the square is 49 square units, what is the radius of the circle?

**A.** $\frac{\sqrt{2}}{7}$ unit
**B.** $\frac{7\sqrt{2}}{2}$ units
**C.** $7\sqrt{2}$ units
**D.** $14\sqrt{2}$ units

24. In circle $B$, $\overline{PQ}$ and $\overline{RS}$ are diameters, and $m\overline{RQ} = 60^\circ$.

![Diagram of circle with labeled segments P, Q, R, S, B, and T.]

- **a.** What is the specific name for quadrilateral $PRQS$?
- **b.** What is the measure of $\angle RQP$?
- **c.** What is the measure of $\angle SBQ$?
- **d.** Use arc lengths to explain why $m\angle PRS = m\angle PQS$. 
25. **Persevere in solving problems.** In circle $S$, chords $\overline{AC}$ and $\overline{BD}$ intersect at $E$. Chord $\overline{AB}$ forms a $21^\circ$ angle with chord $\overline{AC}$ and $m\angle AEB = 140^\circ$.

![Diagram of circle with chords AC, BD intersecting at E, and angle AEB labeled as 140°.]

a. What is $m\widehat{BC}$?

b. What is $m\angle BSC$?

c. What is $m\angle ABE$?

d. What is $m\widehat{AD}$?

e. $\angle AED \cong \angle BEC$ because they are vertical angles. What is the measure of each angle?

**LESSON 25-3**

26. In the circle shown, $m\widehat{XW} = (7x - 10)^\circ$, $m\widehat{ZX} = (11x + 10)^\circ$, $m\widehat{ZY} = 7x^\circ$, and $m\widehat{YW} = 135^\circ$.

![Diagram of circle with chords XW, ZX, YZ, and YW intersecting at T.]

a. Find $x$.

b. Find $m\widehat{XW}$, $m\widehat{ZX}$, and $m\widehat{ZY}$.

c. What is $m\angle XTW$?

d. If $ZT = y + 7$, $TW = 3$, $YT = 5$, and $TX = 2y$, find $ZT$.

27. **Reason quantitatively.** In the circle shown, $m\widehat{BP} = 61^\circ$, $m\angle QMP = 110^\circ$, and $m\widehat{AB} = m\widehat{QP}$.

![Diagram of circle with chords BP, QP intersecting at M, and angle QMP labeled as 110°.]

a. Find $m\angle AMQ$.

b. Find $m\widehat{AQ}$.

c. What is $m\widehat{AB}$?

d. What is $m\angle BAP$?

28. In circle $A$, $\overline{BC}$ is a diameter, $\overline{BE}$, $\overline{ED}$, and $\overline{DC}$ are chords, $m\widehat{BE} = 65^\circ$, and $m\angle BQC = 117.5^\circ$.

![Diagram of circle with diameter BC, chords BE, ED, and DC intersecting at E, Q, and D.]

a. What is $m\angle BEC$?

b. Find $m\widehat{ED}$.

c. Find $m\widehat{DC}$.

d. In $\triangle QED$, is $\overline{QE} \cong \overline{QD}$? Explain.
29. Suppose chords $AB$ and $CD$ intersect at point $E$ inside the circle. Which of the following CANNOT be true?
   A. $\overset{\frown}{AD}$ and $\overset{\frown}{CB}$ can be congruent.
   B. $\overset{\frown}{AC}$ and $\overset{\frown}{DB}$ can be supplementary.
   C. $\overset{\frown}{AC}$ and $\overset{\frown}{BC}$ can be complementary.
   D. $\overset{\frown}{AD}$ and $\overset{\frown}{BC}$ can form a semicircle.

30. Construct viable arguments. In circle $P$, diameter $RS$ is parallel to chord $MN$. Chords $RN$ and $MS$ intersect at point $T$. Tell whether each statement is always, sometimes, or never true.

   a. Figure $RSNM$ is a parallelogram.
   b. Figure $RSNM$ is an isosceles trapezoid.
   c. $\angle RTS$ is obtuse.
   d. $\angle RSM$ and $\angle SNM$ are supplementary.

31. Express regularity in repeated reasoning. In the diagram shown, $\overline{TA}$ and $\overline{TB}$ are tangent to circle $C$.

   a. If $m\overset{\frown}{AB} = 80^\circ$, find $m\overset{\frown}{ADB}$ and $m\angle T$.
   b. If $m\overset{\frown}{ADB} = 210^\circ$, find $m\overset{\frown}{AB}$ and $m\angle T$.
   c. If $m\angle T = 50^\circ$, find $m\overset{\frown}{ADB}$ and $m\overset{\frown}{AB}$.
   d. If $m\angle T = m\overset{\frown}{AB}$, find $m\overset{\frown}{ADB}$, $m\overset{\frown}{AB}$, and $m\angle T$.

32. In circle $O$, $m\overset{\frown}{AB} = (8y + 5)^\circ$, $m\overset{\frown}{BC} = (3y - 1)^\circ$, $m\overset{\frown}{CD} = (4y + 13)^\circ$, and $m\overset{\frown}{AD} = (5y + 3)^\circ$.

   a. What is the value of $y$?
   b. Find $m\overset{\frown}{BC}$.
   c. Find $m\overset{\frown}{AD}$.
   d. What is the measure of $\angle P$?
33. **Persevere in solving problems.** In the diagram shown, $PS$ is tangent to circle $P$ and intersects circle $Q$ at $R$ and $S$. $PX$ is tangent to circle $P$ at $T$ and intersects circle $Q$ at $V$ and $X$. The two circles are tangent at point $Y$. Also, $mSX = 125^\circ$ and $m\angle P = 46^\circ$.

![Diagram](image)

a. What is $m\angle RV$?

b. Find $m\angle SZX$.

c. What is $m\angle QYT$?

d. What is $m\angle QT$?

e. What is $m\angle QPT$?

34. In the diagram, $AB$ is tangent to circle $P$ and $AD$ is a secant.

![Diagram](image)

a. If $mBD = 110^\circ$ and $mBC = 80^\circ$, find $mCD$ and $m\angle A$.

b. If $m\angle A = 51^\circ$ and $mBC = 92^\circ$, find $mBD$ and $mCD$.

c. If $mCD = 135^\circ$ and $mBC = 81^\circ$, find $mBD$ and $m\angle A$.

d. If $m\angle A = 42^\circ$ and $mCD = 160^\circ$, find $mBC$ and $mBD$.

35. The phrase "the measure of the angle is half the difference of the intercepted arcs" applies to all EXCEPT

A. an angle formed by two tangents.
B. an angle formed by two chords.
C. an angle formed by two secants.
D. an angle formed by a secant and a tangent.
**LESSON 26-1**

36. Which statement describes the two steps necessary to prove that point $Q$ is the midpoint of $AB$?

A. Show that $AQ = QB$; show that $AQ = 2 \cdot AB$ and $BQ = 2 \cdot AB$.

B. Show that $AQ = QB$; show that $AQ = \frac{1}{2} AB$ and $BQ = \frac{1}{2} AB$.

C. Show that $A$, $Q$, and $B$ are collinear; show that $AQ + QB = AB$.

D. Show that $AQ = QB$; show that $\angle QAB \cong \angle QBA$.

37. Suppose that points $M$ and $N$ are on a horizontal line, the coordinates of point $M$ are $(x_1, y_1)$, and the midpoint of $MN$ is $T$.

a. Which ordered pair can you use for point $N$, $(x_1, y_2)$ or $(x_2, y_1)$? Explain.

b. What are the coordinates of point $T$?

c. Use the Distance Formula to represent the lengths $MT$ and $TN$.

d. Use the Distance Formula to represent $MN$.


38. **Express regularity in repeated reasoning.** Find the coordinates of the midpoint of $RS$ for each pair of coordinates.

a. $R(2a, 2b), S(2c, 2d)$

b. $R(-4a, -6b), S(4a, 6b)$

c. $R(p, q), S(t, r)$

d. $R(a + 3b, 3a - b), S(3a + 5b, 5a + 7b)$

39. **Reason abstractly.** The center of a circle is $C(a, b)$ and one endpoint of a diameter is $D(2a, 2b)$.

a. Find the coordinates of the other endpoint of that diameter.

b. The diameter is divided into four congruent segments. Find the coordinates of the 5 points that determine those four congruent segments.

40. A rectangle has one vertex at $(0, 0)$ and its diagonals intersect at the point $(m, n)$.

a. What are the coordinates for the other three vertices of the rectangle?

b. What is the distance from any vertex to $(m, n)$?

c. What is the perimeter of the rectangle?

d. What is the area of the rectangle?

**LESSON 26-2**

41. Which statement about the slope of a line is NOT true?

A. A line that goes up from left to right has a positive slope.

B. A line that goes down from left to right has a negative slope.

C. A horizontal line has a slope of zero.

D. A vertical line has a slope of 1.

42. Which of the following statements is true?

A. Two horizontal lines cannot be parallel to each other.

B. Every horizontal line is perpendicular to any vertical line.

C. Two vertical lines can be perpendicular to each other.

D. A line with a positive slope can be parallel to a line with a negative slope.
43. In the diagram, $\overline{AC} \parallel \overline{DE}$ and $\overline{AD}$ is a vertical line. Complete the steps to show that parallel lines have equal slopes.

a. Why is $\angle 1 \cong \angle 2$?

b. Why is $\angle AFC \cong \angle DFE$?

c. What does $\frac{AF}{FC}$ represent?

d. What does $\frac{DF}{FE}$ represent?

e. Why do $\overline{AC}$ and $\overline{DE}$ have equal slopes?

44. Reason abstractly. In the diagram, line $m$ goes through $(0, 0)$ and $(a, b)$, and line $n$ goes through $(-b, a)$ and $(0, 0)$.

a. What is the relationship between $\triangle DCB$ and $\triangle EAC$? Explain.

b. What is the relationship between $\angle 2$ and $\angle 3$? Explain.

c. What is $m\angle BCA$? Explain.

d. Find the slopes of lines $m$ and $n$. Show your work.

e. Find the product of the slopes of lines $m$ and $n$. Show your work.

45. Express regularity in repeated reasoning. Two lines $p$ and $q$ are perpendicular. Describe line $q$ for each description of line $p$.

a. Line $p$ is horizontal.

b. Line $p$ has a negative slope.

c. Line $p$ goes up from left to right.

d. Line $p$ has an undefined slope.

e. Line $p$ goes down from left to right.
LESSON 26-3

Use the diagram for Items 46–49. These items take you through a confirmation that the three medians of a specific triangle are concurrent.

46. The first step is to find the midpoints of the sides.
   a. What are the coordinates of X, the midpoint of BC? Show your work.
   b. What are the coordinates of Y, the midpoint of AC? Show your work.
   c. What are the coordinates of Z, the midpoint of AB? Show your work.
   d. What is the effect of having even integers for the coordinates of points A, B, and C?

47. The second step is to write an equation for each median.
   a. Find the slopes of AX, BY, and CZ. Show your work.
   b. Use the slopes and points A, B, and C to write equations for each median in point-slope form.

48. Attend to precision. The third step is to find the point where two medians intersect.
   a. Using the equations for AX and BY, solve each equation for y and set them equal to each other.
   b. Using your equation from Part a, solve for x. (Hint: You can multiply both sides by a value to remove the fractions.) Show your work.
   c. Using the value for x from Part b and the equation for AX or BY, find the corresponding y-value for the point of intersection of the medians. Show your work.
   d. Write the coordinates of the point of intersection of AX and BY.

49. Make use of structure. The last step is to show that the intersection of two medians is a point that is on the third median.
   a. Show that the point of intersection of AX and BY is on CZ. Show your work.
   b. Summarize what you did in Items 46–49.
50. A student is proving that the medians of $\triangle DEF$ are concurrent. So far, the student has found equations for the three medians $DK$, $EJ$, and $FH$. Which can be the next steps in the student’s proof?

A. Find the slopes of two medians, and show that the product of the slopes is $-1$.
B. Decide whether the three medians intersect inside, on, or outside the triangle, and illustrate each of those with a separate diagram.
C. Find the point of intersection for $FH$ and $EJ$, and then show that $DK$ contains that point.
D. Find the point of intersection for $FH$ and $EJ$, and then find the distance from that point to the three vertices of the triangle.

**LESSON 26-4**

51. **Model with mathematics.** For each pair of ordered pairs, $X$ and $Y$, find the coordinates of a point that lies $\frac{3}{4}$ of the way from $X$ to $Y$.

a. $X(0, 0), Y(20, 28)$

b. $X(5, 1), Y(13, 25)$

c. $X(10, 1), Y(2, -3)$

d. $X(5, -3), Y(11, -18)$

52. In each set of three ordered pairs, $A$ and $B$ are the endpoints of a segment and $P$ is a point on that segment. Show that $\overline{AB}$ and $\overline{AP}$ have the same slope, and then find the ratio $AP : AB$.

a. $A(2, 8), B(8, 10), P(5, 9)$

b. $A(-3, 9), B(5, -7), P(3, -3)$

53. Points $X$, $T$, and $Y$ are on a line segment. Which of the following statements is NOT correct?

A. $T$ is $60\%$ of the distance from $X$ to $Y$.
B. The ratio $TX : TY$ is $3 : 2$.
C. The ratio $TY : XY$ is $3 : 5$.
D. $TX$ and $YT$ have the same slope.

54. Point $H$ lies along a directed line segment from $J(5, 8)$ to $K(1, 1)$. Point $H$ partitions the segment into the ratio $7 : 3$. Find the coordinates of point $H$.

55. **Attend to precision.** Find the coordinates of point $M$ that divides the directed line segment from $P(-1, 3)$ to $Q(9, 8)$ and partitions the segment into the ratio $4$ to $1$. 

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LESSON 27-1

56. A circle has the equation \((x + 5)^2 + (y - 8)^2 = 25\).
   a. Find two points on the circle that have \(x\)-coordinate \(-1\).
   
   b. Find two points on the circle that have \(y\)-coordinate 4.

57. Express regularity in repeated reasoning. Write an equation for each circle.
   a. center \((5, 0)\) and radius 11
   
   b. center \((2, -3)\) and diameter 12
   
   c. center \((0, -4)\) and contains the point \((6, -4)\)
   
   d. diameter has endpoints \((-5, 2)\) and \((10, 10)\)

58. Make use of structure. Identify the center and radius for each circle.
   a. \((x - 3)^2 + (y - 2)^2 = 1\)
   
   b. \((x + 7)^2 + (y + 11)^2 = 10\)
   
   c. \((x - 3.8)^2 + (y - 1.2)^2 = 25\)
   
   d. \((x - a)^2 + (y + b)^2 = m\)

59. A circle contains points \(A(2, 1), B(6, 1)\), and \(C(6, 5)\).
   
   a. Write the equation for the perpendicular bisector of \(\overline{AB}\).
   
   b. Write the equation for the perpendicular bisector of \(\overline{BC}\).
   
   c. Using \(T\) to label the intersection of the lines in Parts \(a\) and \(b\), what are the coordinates of \(T\)?
   
   d. Find the distance from point \(T\) to each of points \(A, B,\) and \(C\).
   
   e. Write an equation for the circle with center \(T\) and radius \(TA\).

60. A circle is drawn on a coordinate grid. Which statement is true?
   A. Every line with a positive slope must either intersect the circle or be tangent to it.
   B. Any line must meet the circle at either 0, 1, or 2 points.
   C. The distance between two points on the circle can never be greater than the length of the circle's radius.
   D. If two points are inside the circle, the distance between them can be greater than the length of the circle's diameter.
LESSON 27-2

61. Add a term so that each expression is the square of a binomial. Then write the new expression in the form \((x + a)^2\).
   a. \(x^2 + 6x\)
   b. \(x^2 - 18x\)
   c. \(y^2 + 5y\)
   d. \(y^2 + 15y\)
   e. \(x^2 + 2a\)

62. Model with mathematics. Write each equation in the form \((x + a)^2 + (y + b)^2 = c\). Then tell what number you added to each side of the original equation.
   a. \(x^2 + 6x + y^2 + 4y = 0\)
   b. \(x^2 - 4x + y^2 + 10y = 0\)
   c. \(x^2 + 2x + y^2 - 5y = 1\)
   d. \(x^2 - 9x + y^2 = 5\)

63. Find the center and radius of the circle represented by each equation.
   a. \((x + 5)^2 + y^2 + 12y = 3\)
   b. \(x^2 + 8x + y^2 - 2y = 8\)
   c. \(x^2 - 20x + y^2 - 6y = -9\)
   d. \(x^2 + x + y^2 - y = \frac{1}{2}\)

64. Construct viable arguments. Determine if the equation represents a circle. If it does, tell the center of the circle.
   a. \(x^2 + 6x + y^2 - 7y = 1\)
   b. \(x^2 - 7x + 5y = 35\)
   c. \(x^2 + y^2 + 3y = 5\)
   d. \((x - 5)^2 + (2y + 3) = 8\)

65. The equation \(x^2 + 3x = y^2 - 4y = 6\) represents a circle. In which quadrant is the center of the circle?
   A. Quadrant I
   B. Quadrant II
   C. Quadrant III
   D. Quadrant IV

LESSON 28-1

66. In the diagram, \(T\) is the point \((0, t)\) and \(m\) is a horizontal line. Which expression represents the distance between point \(T\) and any point \((x, y)\) on line \(m\)?
   A. \(x^2 + (y - t)^2\)
   B. \(\sqrt{x^2 + (y - t)^2}\)
   C. \((x - t)^2 + y^2\)
   D. \(\sqrt{x = (y - t)^2 + y^2}\)
67. A parabola opens up or down. Write the equation of the parabola for the given information.
   a. focus (0, 5), directrix \( y = 5 \)
   b. focus (0, 3), directrix \( y = -3 \)
   c. vertex (0, 0), focus (0, -2)
   d. directrix \( y = -6 \), vertex (0, 6)

68. **Model with mathematics.** Write the focus and directrix for each parabola.
   a. opens to the left; focus (-7, 0); directrix \( x = 7 \)
   b. opens down; focus (0, -1); directrix \( y = 1 \)
   c. opens up; focus \( \left(0, \frac{1}{4}\right) \); directrix \( y = -\frac{1}{4} \)
   d. opens to the right; focus (2.5, 0); directrix \( x = -2.5 \)

69. Write an equation for the parabola illustrated in the diagram.

   ![Parabola Diagram]

   **70. Make use of structure.** For each parabola described below, the vertex of the parabola is the origin. Write the coordinates of the focus.
   a. \( y = \frac{1}{20} x^2 \)
   b. \( y = \frac{1}{5} x^2 \)
   c. \( x = -\frac{1}{10} y^2 \)
   d. \( x = \frac{1}{r} y^2 \)

**LESSON 28-2**

71. What is the vertex of each parabola?
   a. \( y - 5 = \frac{1}{8} (x - 3)^2 \)
   b. \( y = \frac{1}{10} (x + 5)^2 \)
   c. \( x - 3 = \frac{1}{16} (y + 2)^2 \)
   d. \( y = \frac{1}{t} (x + a)^2 + b \)
72. **Construct viable arguments.** Complete the steps to write the equation for the parabola with vertex \((-1, 5)\) and directrix \(y = -1\).

![Parabola diagram]

**a.** Describe how to find the vertex for the parabola. Then find the vertex.

**b.** Explain how to find \(p\) for the parabola. Then find \(p\).

**c.** Tell which way the parabola opens, and explain how you know.

**d.** Write the general form for the equation of the parabola. Then give the values of \(h\), \(k\), and \(p\).

**e.** Use the information in Part d to write the equation for the parabola.

73. **Make use of structure.** For each parabola,

(i) Determine the direction of the opening.

(ii) Determine whether the general form for the parabola is \(y - k = \frac{1}{4p} (x - h)^2\) or \(y - h = \frac{1}{4p} (y - k)^2\).

(iii) Give the values of \(h\), \(k\), and \(p\) for the parabola.

(iv) Write the equation for the parabola.

**a.** vertex \((-3, 1)\), directrix \(x = -6\)

**b.** focus \((0, 1.5)\), directrix \(y = 2.5\)

74. Which statement describes the vertex and directrix of the parabola \(y - 3 = \frac{1}{16} (x + 5)^2\)?

A. vertex \((-5, 3)\); directrix \(y = -1\)

B. vertex \((-5, 3)\); directrix \(y = 1\)

C. vertex \((3, -5)\); directrix \(y = -9\)

D. vertex \((3, -5)\); directrix \(y = -1\)

75. The equation of a parabola is \(y^2 + 2y + 1 - x = 5\). Find the vertex of the parabola. Explain your steps.

**LESSON 29-1**

76. **Use appropriate tools strategically.** Use quadrilateral PQRS. Do not erase your construction marks.

![Quadrilateral diagram]

**a.** Using \(\overline{OA}\), construct \(\overline{OB}\) so \(OB = PQ\).

**b.** Draw \(\overline{CD}\). Then identify \(\overline{XY}\) on \(\overline{CD}\), so \(XY = SR\).

**c.** Construct a segment whose length is \(PS - SR\). Label that segment "PS - SR."

**d.** Construct a segment whose length is three times \(\overline{PQ}\).
77. Use \( \triangle ABC \). Do not erase your construction marks.

\[ \begin{array}{c}
A \\
B \\
C \\
X \\
Y \\
\end{array} \]

a. Using \( XY \), copy angle \( B \) at point \( X \).

b. Point \( P \) is on \( DE \). Copy angles \( A \), \( B \), and \( C \) so they are non-overlapping adjacent angles and each has vertex \( P \).

78. Construct the perpendicular bisector of each given segment. Do not erase your construction marks.

a. \[ \begin{array}{c}
M \\
N \\
\end{array} \]

b. \[ \begin{array}{c}
R \\
\end{array} \]

79. **Attend to precision.** Construct the angle bisectors for each angle of \( \triangle RST \).

80. One side of \( \triangle XYZ \) has a length of 17 cm. Which pairs of lengths CANNOT be the lengths of the other two sides of the triangle?

A. 1 cm, 17 cm  
B. 35 cm, 19 cm  
C. 25 cm, 5 cm  
D. 10 cm, 10 cm

**LESSON 29-2**

81. **Use appropriate tools strategically.** Through point \( H \), construct a line \( p \) that is parallel to line \( m \). Then, through point \( Q \), construct a line \( q \) that is also parallel to line \( m \).

\[ \begin{array}{c}
H \\
\end{array} \]

82. Construct a line \( r \) that contains point \( C \) and is perpendicular to \( \overline{AB} \). Then select a point \( D \) on line \( r \) and, through \( D \), construct a line \( s \) that is perpendicular to line \( r \).

\[ \begin{array}{c}
B \\
A \\
C \\
\end{array} \]
83. In the diagram, point \( M \) is on \( HK \) and point \( N \) is not on \( HK \).

\[ H \quad M \quad N \quad K \]

a. Construct line \( a \) through point \( N \) so that \( a \perp HK \).

b. Construct line \( b \) through point \( M \) so that \( b \perp HK \).

c. What is the relationship between lines \( a \) and \( b \)?

d. Construct line \( c \) through point \( N \) so that \( c \perp a \).

e. What is the relationship between line \( c \) and \( HK \)?

84. **Reason abstractly.** Points \( R \), \( S \), and \( T \) are on a circle.

\[ R \quad S \quad T \]

a. Draw \( RS \). Then construct line \( m \) so it is the perpendicular bisector of \( RS \).

b. Draw \( ST \). Then construct line \( n \) so it is the perpendicular bisector of \( ST \).

c. How is the intersection of \( m \) and \( n \) related to the circle?

d. Use a compass to verify your answer to Part c.

85. What construction is shown in the diagram?

\[ \text{A. finding the perpendicular bisectors of the three sides of a triangle} \]

\[ \text{B. finding the bisectors of the three angles of a triangle} \]

\[ \text{C. finding the medians to the three sides of a triangle} \]

\[ \text{D. finding the altitudes to the three sides of a triangle} \]

86. **Use appropriate tools strategically.** \( AB \) is the radius of a circle. Construct a circle with that radius. Then construct a regular inscribed hexagon in the circle.

\[ A \quad B \]

87. \( PR \) is a diagonal of square \( PQRS \). Construct that square.
88. **Attend to precision.** Follow these steps to inscribe a circle in \( \triangle MNP \).

![Diagram of \( \triangle MNP \)]

a. Bisect \( \angle N \). Use \( s \) to label the bisector.

b. Find the intersection of the bisectors of \( \angle N \) and \( \angle P \). Use \( T \) to label that point.

c. Construct line \( z \) through point \( T \) so that \( z \) is perpendicular to \( NP \). Use \( Q \) to label the intersection of line \( z \) and \( NP \).

d. Using \( T \) as the center and \( TQ \) as a radius, construct circle \( T \).

89. Point \( B \) is a point on circle \( A \).

![Diagram of circle with point \( B \)]

a. Construct line \( t \) that is tangent to circle \( A \) at point \( B \). Explain your steps.

b. Construct circle \( P \) that is tangent to circle \( A \). Circle \( P \) should have a center that is on \( AB \) and a radius equal to the radius of circle \( A \). Explain your steps.

90. A student has constructed square \( PRTV \) inscribed in circle \( N \). The student wants to inscribe a regular octagon in the circle. Which construction will NOT result in the other four vertices of the octagon?

![Diagram of circle with square \( PRTV \)]

A. Using \( P \) as the center and \( \overline{PN} \) as a radius, draw arcs on the circle on each side of \( P \). Repeat using \( T \) as the center.

B. Construct the perpendicular bisectors of \( PV \) and \( PR \), and identify the four points where the perpendicular bisectors intersect the circle.

C. Draw diameters \( PT \) and \( VR \) for the circle. Bisect the four central angles, and identify the points where the angle bisectors intersect the circle.

D. Construct perpendiculars from point \( N \) to each of the four sides of the square. Identify the points where the perpendiculars intersect the circle.